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γ_5 in the four-dimensional helicity scheme

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We investigate the regularization-scheme dependent treatment of γ_5 in the framework of dimensional regularization, mainly focusing on the four-dimensional helicity scheme (FDH). Evaluating distinctive examples, we find that for one-loop calculations, the recently proposed four-dimensional formulation (FDF) of the FDH scheme constitutes a viable and efficient alternative compared to more traditional approaches. In addition, we extend the considerations to the two-loop level and compute the pseudoscalar form factors of quarks and gluons in FDH. We provide the necessary operator renormalization and discuss at a practical level how the complexity of intermediate calculational steps can be reduced in an efficient way.

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I. INTRODUCTION

The success of quantum-field theoretical predictions over the past decades was enabled, among other things, by the applicability of dimensional regularization as the method provides a mathematically consistent tool to handle ultraviolet (UV) and infrared (IR) divergences in the multi-loop regime. From the very moment of the introduction of dimensional regularization in Ref. [1], however, special attention had to be paid to the treatment of γ_5 since the object is closely related to concepts that are only valid in integer dimensions. In a series of publications [2–15] that cover a time span of more than 40 years, different approaches have been developed in order to find consistent rules for the treatment of γ_5 in the dimensional framework. Irrespective of this effort, in the overwhelming majority of computations that have been performed so far, the original γ_5 definition of Ref. [1] has been used, giving expression to the fact that even today no efficient alternatives are available that are well suited for all kinds of calculations.

Parallel to the development of γ_5 schemes, the search for new efficient calculational methods has focused on finding regularization prescriptions that reduce the technical complexity at the practical level. Recently, the current status of the most prominent schemes has been summarized in Ref. [16]. Among the considered dimensional schemes are the 't Hooft-Veltman scheme (HV) [1], conventional dimensional regularization (CDR) [17], dimensional reduction (DRED) [18], the four-dimensional helicity scheme (FDH) [19,20], and its recently proposed four-dimensional formulation (FDF) [21] at one loop.

In this article, we investigate the treatment of γ_5 in the aforementioned dimensional schemes, mainly concentrating on the FDH scheme. As prescriptions for γ_5 we consider the original one of 't Hooft/Veltman and an anticommuting γ_5. Having the practitioner in mind, we perform distinctive one- and two-loop calculations and show which of the γ_5 schemes is the more efficient alternative for the respective process under consideration. In order to enable a step-by-step comparison between the different γ_5 schemes and the different dimensional schemes, the outline of the letter is the following: In Sec. II A, we provide the definitions of γ_5 in CDR/HV and extend them to FDH/DRED in Sec. II B. To illustrate practical consequences of these definitions, we evaluate characteristic one-loop examples in Secs. II C and II D, putting emphasis on differences and similarities of the various approaches. The extension of these considerations to the two-loop level is discussed in Sec. III by computing the pseudo-scalar form factors of quarks and gluons in massless QCD. The necessary operator renormalization as well as the UV-renormalized results are provided in Sec. IV.

II. TREATMENT OF γ_5 IN DIMENSIONAL REGULARIZATION

A. CDR and HV

One main reason for the recurrent appearance of seeming inconsistencies related to γ_5 is the fact that for a consistent

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formulation of d-dimensional integration, the four-dimensional Minkowski space $S_{[4]}$ has to be embedded into an *infinite*-dimensional space $\mathrm{QS}_{[d]}$[1] [17],

$$S_{[4]} \subset \mathrm{QS}_{[d]}. \tag{2.1}$$

Although $\mathrm{QS}_{[d]}$ and the related quantities formally have finite-dimensional properties, common concepts of $S_{[4]}$ like index counting are no longer applicable. Regarding γ_5, this interplay between finite- and infinite-dimensional aspects has caused quite a lot of confusion in the past and led to the introduction of different γ_5 schemes (GS).

Depending on which GS is chosen, special attention has to be paid to the evaluation of the Lorentz algebra, to the breaking of symmetries, to the treatment of anomalies, and to the UV renormalization at higher perturbative orders. According to the different characteristics regarding these points, it is useful to distinguish the following two classes of GS:

(i) The first class contains schemes where γ_5 is defined by a *construction prescription* like in the original definition by 't Hooft/Veltman [1] and Breitenlohner/Maison (BM)[2] [2],

$$\underline{\mathrm{BM}}\colon\ \gamma_5^{\mathrm{BM}} \equiv \frac{i}{4!}(\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)_{[4]} \equiv \frac{i}{4!}\varepsilon^{\mu\nu\rho\sigma}_{[4]}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma)_{[d]}. \tag{2.2}$$

(ii) The second class contains schemes where γ_5 is defined *algebraically*, for example as anticommuting (AC) with (quasi) d-dimensional γ matrices [4,5],

$$\underline{\mathrm{AC}}\colon\ \{\gamma_5^{\mathrm{AC}}, \gamma^\mu_{[d]}\} \equiv 0. \tag{2.3}$$

In Eq. (2.2), γ_5^{BM} is defined via the totally antisymmetric Levi-Civita pseudotensor $\varepsilon^{\mu\nu\rho\sigma}$ which is closely related to the concept of index counting in strictly four dimensions. While the dimensionality of the γ matrices is treated differently in various dimensional schemes, it is mandatory to consider $\varepsilon^{\mu\nu\rho\sigma}$ as a *strictly* four-dimensional object. Only in this way it is possible to avoid ambiguous results and mathematical inconsistencies found before e. g. in Ref. [22]. Usually, the mismatch between the dimensionality of $\varepsilon^{\mu\nu\rho\sigma}$ and other algebraic objects is circumvented by workarounds whose ranges of validity are often not obvious, at least not at first sight. More details regarding this issue will be given in Sec. III B.

A direct consequence of Eq. (2.2) is that all (anti) commutation relations of γ_5^{BM} are implicitly part of the definition and therefore fixed, e.g.

$$\{\gamma_5^{\mathrm{BM}}, \gamma^\mu_{[4]}\} = 0, \qquad [\gamma_5^{\mathrm{BM}}, \gamma^\mu_{[d-4]}] = 0, \tag{2.4a}$$

and therefore [2]

$$\{\gamma_5^{\mathrm{BM}}, \gamma^\mu_{[d]}\} = 2\gamma^\mu_{[d-4]}\gamma_5^{\mathrm{BM}}. \tag{2.4b}$$

It is clear that Eqs. (2.3) and (2.4b) yield different results for $d \neq 4$, at least at intermediate steps of the calculation. In the UV renormalized (and IR subtracted) theory, however, different consistent approaches have to yield the same results for physical observables.

B. FDH and DRED

So far, the algebraic behavior of γ_5 has been considered in the quasi d-dimensional space $\mathrm{QS}_{[d]}$ which is the natural domain of CDR and of d-dimensional integration. In Ref. [23], it is shown that in order to consistently formulate FDH and DRED, this space has to be enlarged to $\mathrm{QS}_{[d_s]}$ via a direct (orthogonal) sum with the so-called "evanescent" space $\mathrm{QS}_{[n_\epsilon]}$,

$$\mathrm{QS}_{[d_s]} \equiv \mathrm{QS}_{[d]} \oplus \mathrm{QS}_{[n_\epsilon]}. \tag{2.5}$$

Although d_s is usually taken to be 4 in FDH and DRED, it is clear that $\mathrm{QS}_{[d_s]}$ is an infinite-dimensional space with finite-dimensional algebraic properties.[3]

According to the structure of the vector spaces in Eq. (2.5), quasi d_s-dimensional metric tensors and γ matrices can be split as $g^{\mu\nu}_{[d_s]} = g^{\mu\nu}_{[d]} + g^{\mu\nu}_{[n_\epsilon]}$ and $\gamma^\mu_{[d_s]} = \gamma^\mu_{[d]} + \gamma^\mu_{[n_\epsilon]}$, resulting in

$$(g_{[dim]})^\mu{}_\mu = dim, \qquad (g_{[d]}g_{[n_\epsilon]})^\mu{}_\nu = 0, \tag{2.6a}$$

$$\{\gamma^\mu_{[dim]}, \gamma^\nu_{[dim]}\} = 2g^{\mu\nu}_{[dim]}, \qquad \{\gamma^\mu_{[d]}, \gamma^\nu_{[n_\epsilon]}\} = 0, \tag{2.6b}$$

with $dim \in \{4, d, d_s, n_\epsilon\}$.

As mentioned before, the (anti)commutation relations of γ_5^{BM} are fixed by Eq. (2.2), e.g.

$$\underline{\mathrm{BM}}\colon\ \{\gamma_5^{\mathrm{BM}}, \gamma^\mu_{[d]}\} = 2\gamma^\mu_{[d-4]}\gamma_5^{\mathrm{BM}}, \qquad [\gamma_5^{\mathrm{BM}}, \gamma^\mu_{[n_\epsilon]}] = 0. \tag{2.7a}$$

Due to the even number of γ matrices in Eq. (2.2), γ_5^{BM} *commutes* with the evanescent degrees of freedom in FDH and DRED. Moreover, from Eq. (2.7a) it directly follows that the structure of the (anti)commutation relation in d and d_s dimensions is the same,

[1]Following Ref. [16], we denote the (quasi)dimensionality *dim* of a quantity by a subscript [*dim*]. Throughout this article, the modified space-time dimension is always defined as $d \equiv 4 - 2\epsilon$.

[2]In order to distinguish this prescription from other aspects of the original HV scheme, we solely use the abbreviation BM to denote a scheme for the treatment of γ_5.

[3]For more comments on the definition and the structure of the vector spaces in Eq. (2.5) we refer to [23–25] and references therein. Here it should only be mentioned that setting $d_s = 4$ results in $n_\epsilon = 2\epsilon$.

$$\{\gamma_5^{\rm BM}, \gamma^\mu_{[d_s]}\} = 2\gamma^\mu_{[d_s-4]}\gamma_5^{\rm BM}. \tag{2.7b}$$

As a consequence, in practical calculations it is possible to either use a quasi d_s-dimensional Lorentz algebra or to explicitly perform the split of Eq. (2.5).

In contrast, the (anti)commutation relations of $\gamma_5^{\rm AC}$ are not fixed *a priori* but have to be part of the definition. We therefore *define*

$$\underline{\rm AC}\colon\ \{\gamma_5^{\rm AC}, \gamma^\mu_{[d]}\} \equiv 0, \qquad \{\gamma_5^{\rm AC}, \gamma^\mu_{[n_\epsilon]}\} \equiv 0, \tag{2.8a}$$

resulting in

$$\{\gamma_5^{\rm AC}, \gamma^\mu_{[d_s]}\} = 0. \tag{2.8b}$$

At first sight, it might seem appropriate to use a *commutator* in the right definition of Eq. (2.8a), in a similar way as in Eq. (2.7a). In general, however, calculations in FDH and DRED are significantly facilitated if one uses a quasi d_s-dimensional algebra instead of performing the split in Eq. (2.5). This option is guaranteed by Eq. (2.8) since the algebra in d and d_s dimensions is the same. Moreover, in Secs. II C and IV B it will be shown that exclusively using anticommutators in Eq. (2.8) results in a much simpler UV renormalization. It is also a convenient choice regarding the nonbreaking of supersymmetry [23].

To illustrate the implications of the different schemes for γ_5, we consider the following simple one-loop examples in the FDH scheme: the correlator $\gamma^\mu\gamma_5 \to e^+e^-$ and the (anomalous) correlator of an axial-vector current and two vector currents (AVV correlator). Each of the examples is evaluated by using $\gamma_5^{\rm BM}$ and $\gamma_5^{\rm AC}$ as defined in Eqs. (2.2) and (2.3), respectively. In addition we apply FDF, a recently proposed genuine four-dimensional formulation of the FDH algebra at the one-loop level. In the analytical results, the fermion mass is denoted by m and p_1, p_2 are the (outgoing) momenta of the external fermions/gauge fields. For simplicity we consider QED and set $e = 1$ for the gauge coupling.

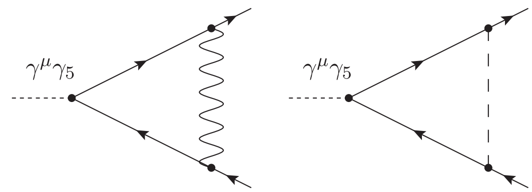

FIG. 1. One-loop contributions to the correlator $\gamma^\mu\gamma_5 \to e^+e^-$. The diagrams contain a gauge field (left) and an associated FDF-scalar (right). The latter diagram is only present in FDF.

C. One-loop example 1: Correlator $\gamma^\mu\gamma_5 \to e^+e^-$

1. FDH and $\gamma_5^{\rm BM}$

The application of $\gamma_5^{\rm BM}$ in a d-dimensional framework with $d \neq 4$ results in different algebraic properties compared to the unregularized theory which can be easily seen from Eq. (2.4). The (d-dimensional) axial-vector operator is therefore usually symmetrized "by hand" and written as [6]

$$\gamma^\mu_{[4]}\gamma_5 \to \frac{1}{2}(\gamma^\mu_{[d]}\gamma_5^{\rm BM} - \gamma_5^{\rm BM}\gamma^\mu_{[d]}). \tag{2.9}$$

Using this relation together with Eqs. (2.2) and (2.6), and multiplying with $q_\mu \equiv (p_1 + p_2)_\mu$ then yields for the left diagram in Fig. 1[4]

$$\begin{aligned} q_\mu T^\mu\Big|_{\rm bare} &\to \frac{\varepsilon^{\mu\nu\rho\sigma}_{[4]}}{2\times 4!}\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\alpha_{[d_s]}[(\not{k}+\not{p}_1+m)(\not{q}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma - \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\not{q})(\not{k}-\not{p}_2+m)]_{[d]}\gamma^\beta_{[d_s]}(g_{\alpha\beta})_{[d_s]}}{[(k+p_1)^2_{[d]}-m^2][(k-p_2)^2_{[d]}-m^2]k^2_{[d]}} \\ &= \frac{1}{(4\pi)^2}\left[\frac{1-\frac{n_\epsilon}{2}}{\epsilon}+\frac{9}{2}+\mathcal{O}(\epsilon)+\mathcal{O}(m^2)\right]\not{q}_{[d]}\gamma_5^{\rm BM}. \end{aligned} \tag{2.10}$$

The (on-shell) renormalization of the external fermion fields as well as the prediction for the structure of the IR divergences in the FDH scheme are given in Ref. [26],

$$\delta\bar{Z}_2^{(1)}(n_\epsilon) = \frac{1}{(4\pi^2)}\left[\frac{-3-\frac{n_\epsilon}{2}}{\epsilon} - 4 + \mathcal{O}(\epsilon) + \mathcal{O}(m^2)\right], \tag{2.11a}$$

$$\bar{\mathbf{Z}}_{\rm IR}^{(1)} = \frac{1}{(4\pi^2)}\left[-\frac{2}{\epsilon} + \mathcal{O}(m^2)\right]. \tag{2.11b}$$

Subtracting the IR divergence, it follows that field renormalization is not sufficient to obtain the correct result since the (scheme-dependent) UV divergence does not cancel. The general reason is that symmetries of the unregularized theory like chiral and Lorentz invariance

[4]In this example, Lorentz indices related to vector fields are treated in $d_s = d + n_\epsilon$ dimensions, see also Eq. (2.5). The case $d_s = 4$ (and therefore $n_\epsilon = 2\epsilon$) then corresponds to FDH and DRED, whereas results in CDR and HV are obtained for $n_\epsilon = 0$. Here and in the following, the irrelevant dimension of the external momenta is set to d and terms of $\mathcal{O}(\epsilon^0 n_\epsilon)$ are omitted since they vanish after setting $n_\epsilon = 2\epsilon$ and taking the subsequent limit $\epsilon \to 0$. The regularization scale is fixed via $\mu_0 \equiv m$. Note further, that the ε pseudotensor is considered outside dimensional regularization and treated in strictly four dimensions.

are broken explicitly if $\gamma_5^{\rm BM}$ is used in a d-dimensional framework.[5] As a consequence, initial symmetries have to be restored by means of additional counterterms. In Sec. IV A, it will be shown that for the one-loop example at hand, this renormalization reads

$$\delta\bar{Z}^{{\rm BM},(1)}(n_\epsilon) = \delta\bar{Z}^{{\rm BM},(1)}_{\overline{\rm MS}}(n_\epsilon) + \delta Z_5^{(1)} = \frac{1}{(4\pi)^2}\left[\frac{n_\epsilon}{\epsilon} - 4\right]. \tag{2.12}$$

It is given by a pure $\overline{\rm MS}$ pole term $\delta\bar{Z}^{\rm BM}_{\overline{\rm MS}}$ which is finite after setting $n_\epsilon = 2\epsilon$ and by a regularization-scheme independent constant δZ_5. In CDR ($n_\epsilon = 0$), the latter is usually determined through relations that are valid in strictly four-dimensional schemes like the Pauli-Villars setup, see e.g. Ref. [6]. In Sec. IV we present an alternative approach that is based on a comparison between results obtained with $\gamma_5^{\rm BM}$ and $\gamma_5^{\rm AC}$.

Combining Eqs. (2.10)–(2.12) and taking the subsequent limit $d \to 4$, we obtain for the UV-renormalized and IR-subtracted correlator

$$q_\mu T^\mu = \frac{1}{(4\pi)^2}\left[-\frac{7}{2} + \mathcal{O}(m^2)\right] \not{q}_{[4]}\gamma_5. \tag{2.13}$$

Since all evanescent terms $\sim n_\epsilon$ drop out through UV renormalization, this final result does not depend on the applied dimensional scheme.

2. FDH and $\gamma_5^{\rm AC}$

For the case of an anticommuting $\gamma_5^{\rm AC}$ we write the FDH one-loop amplitude as

$$q_\mu T^\mu\Big|_{\rm bare} \to -i\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\alpha_{[d_s]}[(\not{k}+\not{p}_1+m)\not{q}\gamma_5^{\rm AC}(\not{k}-\not{p}_2+m)]_{[d]}\gamma^\beta_{[d_s]}(g_{\alpha\beta})_{[d_s]}}{[(k+p_1)^2_{[d]}-m^2][(k-p_2)^2_{[d]}-m^2]k^2_{[d]}} = \frac{1}{(4\pi)^2}\left[\frac{1+\frac{n_\epsilon}{2}}{\epsilon} + \frac{1}{2} + \mathcal{O}(\epsilon) + \mathcal{O}(m^2)\right]\not{q}_{[d]}\gamma_5^{\rm AC}. \tag{2.14}$$

The result has been obtained by (anti)commuting $\gamma_5^{\rm AC}$ to the right and evaluating the remaining algebra by means of Eq. (2.6). Due to the absence of an explicit symmetrization and the reduced number of γ matrices in the numerator, the evaluation of the algebra is much simpler compared to Eq. (2.10). Moreover, the consequent use of an anticommutator in Eq. (2.8) leads to a sign change of the n_ϵ term. Applying the field renormalization of Eq. (2.11a) and subtracting the IR divergence we then directly recover the result in Eq. (2.13). In contrast to $\gamma_5^{\rm BM}$ therefore no symmetry-restoring counterterms are needed to get the correct result.

3. Algebra in genuine four dimensions—FDF

FDF is a novel regularization approach that was introduced to reproduce FDH results at the one-loop level [21]. Starting from unregularized analytical expressions, loop momenta in FDF are shifted as $\not{k}_{[4]} \to \not{k}_{[d]} \equiv \not{k}_{[4]} + i\mu\gamma_5$ *before* any other algebraic manipulation is performed. The scale μ corresponds to the $(d-4)$-dimensional components of the loop momentum and serves as a regulator for the in general divergent quasi d-dimensional loop integrals. By definition, odd powers of μ are set to zero, resulting in the useful relation

$$\not{k}_{[d]}\not{k}_{[d]} = k^2_{[d]} = k^2_{[4]} - \mu^2. \tag{2.15}$$

One main advantage of the FDF approach is that the Lorentz algebra is realized in strictly four dimensions; Eqs. (2.2) and (2.3) are therefore equivalent, i.e. $\gamma_5^{\rm BM} = \gamma_5^{\rm AC} \equiv \gamma_5$. Applying this setup, the analytical expression for the left diagram in Fig. 1 reads[6]

$$q_\mu T^\mu|_{\rm bare} \to -i\int\frac{d^dk}{(2\pi)^d}\frac{[\gamma^\alpha(\not{k}+i\mu\gamma_5+\not{p}_1+m)\not{q}\gamma_5(\not{k}+i\mu\gamma_5-\not{p}_2+m)\gamma^\beta g_{\alpha\beta}]_{[4]}}{[(k+p_1)^2_{[d]}-m^2][(k-p_2)^2_{[d]}-m^2]k^2_{[d]}} = \frac{1}{(4\pi)^2}\left[\frac{1}{\epsilon} + \frac{7}{2} + \mathcal{O}(\epsilon) + \mathcal{O}(m^2)\right]\not{q}_{[4]}\gamma_5. \tag{2.16}$$

For the evaluation of the algebra we used Eq. (2.15) to cancel against the denominator, resulting in the μ^2-dependent "extra integral" [16]

[5] In the original reference of 't Hooft/Veltman [1], for example, it is shown how the use of Eq. (2.4b) leads to a breaking of Ward identities. See also Ref. [6] for a pedagogical review.

[6] Using Feynman gauge, the right diagram including a so-called FDF-scalar vanishes according to the rules of FDF; in other gauges, both diagrams in Fig. 1 contribute. In the latter case, the diagrams sum up to the same (gauge-independent) result as given in Eq. (2.16). For more details regarding gauge dependence in FDF we refer to Ref. [16].

$$I_3^d(\mu^2) = \int \frac{d^dk}{(2\pi)^d} \frac{\mu^2}{[(k+p_1)^2_{[d]} - m^2][(k-p_2)^2_{[d]} - m^2]k^2_{[d]}} = \frac{i}{(4\pi)^2}\left[\frac{1}{2} + \frac{3}{2}\epsilon + \mathcal{O}(\epsilon^2)\right] + \mathcal{O}(m^2). \tag{2.17}$$

Although only strictly four-dimensional quantities and an anticommuting γ_5 have been used to obtain the result in Eq. (2.16), the γ_5^{BM} result in Eq. (2.10) for $n_\epsilon = 2\epsilon$ is recovered. The conceptual reason is that within FDF, similar relations as in Eq. (2.4b) hold, e.g.[7]

$$\underline{\mathrm{FDF}}\colon \{\gamma_5, \not{k}_{[d]}\} = 2i\mu. \tag{2.18}$$

To obtain a physical result that is compatible with the symmetries of the underlying theory we therefore have to add the same counterterms as for the case of γ_5^{BM}. Compared to Eq. (2.10), however, the evaluation of the analytical expressions is significantly simplified.

D. One-loop example 2: AVV triangle

As a second example we consider the AVV triangles in Fig. 2 for the case of massless fermions. In the present case of an NLO fermion loop, the only difference between the dimensional schemes CDR, HV, FDH, and DRED is the dimensionality of the external gauge-field momenta. Since the final result of the amplitude is finite, as will be shown below, the limit $d \to 4$ can be taken without any UV renormalization. After having taken the physical limit, the virtual one-loop amplitudes are therefore the same in all these dimensional schemes.

1. FDH and γ_5^{BM}

Applying the same setup as in the previous example we obtain in CDR

$$\begin{aligned} q_\mu T^{\mu\alpha\beta}_{\mathrm{AVV}} &\to \frac{i\varepsilon^{\mu\nu\rho\sigma}_{[4]}}{2\times 4!}\int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}[(\not{q}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma - \gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\not{q})(\not{k}+\not{p}_1)\gamma^\alpha\not{k}\gamma^\beta(\not{k}-\not{p}_2)]_{[d]}}{(k+p_1)^2_{[d]}k^2_{[d]}(k-p_2)^2_{[d]}} + \begin{pmatrix} p_1 \leftrightarrow p_2 \\ \alpha \leftrightarrow \beta \end{pmatrix} \\ &= -\frac{1}{2\pi^2}\varepsilon^{\alpha\beta\mu\nu}_{[4]}\{p_{1,\mu}p_{2,\nu}\}_{[d]}[1 + 3\epsilon + \mathcal{O}(\epsilon^2)], \end{aligned} \tag{2.19}$$

where, as before, the ε pseudotensor is considered outside dimensional regularization throughout the calculation. Taking the limit $\epsilon \to 0$, the result in Eq. (2.19) coincides with the well-known (anomalous) axial Ward identity (AWI) given e.g. in Refs. [27–29].

2. FDH and γ_5^{AC}

One important characteristic related to the treatment of γ_5^{AC} in dimensional schemes is that traces including odd numbers of γ_5^{AC} either vanish or are not cyclic anymore. Demanding, for example, cyclicity of traces including γ_5^{AC} leads to relations like [9]

$$(d-4)\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5^{\mathrm{AC}}]_{[d]} = 0. \tag{2.20}$$

For $d \neq 4$, this equation can only be fulfilled for a vanishing trace. Since similar relations hold for other numbers of γ matrices in the trace we get

$$q_\mu T^{\mu\alpha\beta}_{\mathrm{AVV}} = 0 \tag{2.21}$$

and gauge invariance is broken explicitly. Different solutions have been proposed e.g. in Refs. [4,5] and [12,13,15] by modifying the trace operation in such a way that the result in Eq. (2.19) is recovered. These modified traces, however, lead to significant complications in practical calculations, in particular at higher perturbative orders. In this paper we therefore refrain from the explicit evaluation of γ_5^{AC}-odd traces. Instead, in Sec. IV B we show how this can be avoided at the practical level.

3. FDF

Finally we evaluate the triangle diagrams by utilizing the FDF approach. Using the same four-dimensional Feynman rules as in Sec. II C, the analytical expression reads

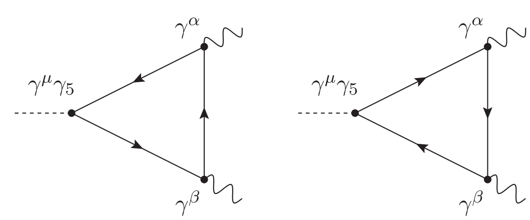

FIG. 2. One-loop contributions to the (anomalous) AVV correlator $T^{\mu\alpha\beta}_{\mathrm{AVV}}$ including one axial-vector and two vector vertices.

[7]This relation follows from $\gamma_5\not{k}_{[d]} = \gamma_5(\not{k}_{[4]} + i\mu\gamma_5) = (-\not{k}_{[4]} + i\mu\gamma_5)\gamma_5 = -\not{k}_{[d]}\gamma_5 + 2i\mu$. It is important to notice that in practical computations, relations like in Eq. (2.18) are not used explicitly since quasi d-dimensional quantities are in FDF split into a strictly four-dimensional and a μ-dependent part. The γ_5 matrix is therefore effectively an anticommuting one.

$$q_\mu T^{\mu\alpha\beta}_{\rm AVV} \to \int \frac{d^dk}{(2\pi)^d} \frac{{\rm Tr}[\not{q}\gamma_5(\not{k}+i\mu\gamma_5+\not{p}_1)\gamma^\alpha(\not{k}+i\mu\gamma_5)\gamma^\beta(\not{k}+i\mu\gamma_5-\not{p}_2)]_{[4]}}{(k+p_1)^2_{[d]}k^2_{[d]}(k-p_2)^2_{[d]}} + \begin{pmatrix} p_1 \leftrightarrow p_2 \\ \alpha \leftrightarrow \beta \end{pmatrix}. \tag{2.22}$$

A crucial difference compared to other dimensional schemes is the appearance of rank two tensor integrals with strictly four-dimensional loop momenta in the numerator,

$$\int \frac{d^dk}{(2\pi)^d} \frac{k^\rho_{[4]}k^\sigma_{[4]}}{(k+p_1)^2_{[d]}k^2_{[d]}(k-p_2)^2_{[d]}} \equiv C_{00}g^{\rho\sigma}_{[4]} + C_{12}(p_1^\rho p_2^\sigma + p_2^\rho p_1^\sigma)_{[4]} + \cdots. \tag{2.23a}$$

Using Eq. (2.15) and neglecting odd powers of μ, the relevant coefficient is given by

$$C_{00} = \frac{1}{2}\left\{\int \frac{d^dk}{(2\pi)^d}\frac{1}{(k+p_1)^2_{[d]}(k-p_2)^2_{[d]}} + \int \frac{d^dk}{(2\pi)^d}\frac{k^2_{[4]}}{(k+p_1)^2_{[d]}k^2_{[d]}(k-p_2)^2_{[d]}}\right\}. \tag{2.23b}$$

The first integrand is given by d-dimensional quantities only and the integral can be evaluated without any complication. In contrast, the second integral contains strictly four-dimensional components of the loop momentum. Using Eq. (2.15) to cancel against the denominator gives rise to the integral in Eq. (2.17) for $m = 0$. It turns out that this integral is the only one that contributes to the AVV correlator in the FDF approach. In other words, the anomaly is entirely given by a μ^2 integral that stems from the evaluation of the tensor integrals,

$$q_\mu T^{\mu\alpha\beta}_{\rm AVV} \to 16iI^d_3(\mu^2)\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]} \tag{2.24a}$$

$$= -\frac{1}{2\pi^2}\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]}[1+3\epsilon+\mathcal{O}(\epsilon^2)]. \tag{2.24b}$$

In this way, the result in Eq. (2.19) is recovered, including higher terms in the ϵ expansion. Again, the computational effort is significantly reduced compared to the case of $\gamma_5^{\rm BM}$.

4. Comment on Bose symmetry

Recently it has been shown [30,31] that special care has to be taken when using an anticommuting $\gamma_5^{\rm AC}$ since gauge invariance and Bose symmetry may not be maintained simultaneously, even if the dimension of the underlying space-time remains unchanged during the regularization process. At the root of this symmetry breaking are $\gamma_5^{\rm AC}$-odd traces which yield different contributions compared to the case of $\gamma_5^{\rm BM}$.

In Ref. [31], the interplay between gauge invariance and Bose symmetry is investigated in the framework of implicit regularization (IREG). Using $\gamma_5^{\rm BM}$ as defined in Eq. (2.2) together with the right- and left-handed chiral operators $V^\mu_{\rm R} \equiv \frac{1}{2}\gamma^\mu(\mathbb{1}+\gamma_5^{\rm BM})$ and $V^\mu_{\rm L} \equiv \frac{1}{2}\gamma^\mu(\mathbb{1}-\gamma_5^{\rm BM})$ at the vertices, the following results for the different correlators are provided[8]

<u>IREG/BM</u>:
$$q_\mu T^{\mu\alpha\beta}_{\rm RRR} = -q_\mu T^{\mu\alpha\beta}_{\rm LLL} = -\frac{1}{12\pi^2}\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]}, \tag{2.25a}$$

$$q_\mu T^{\mu\alpha\beta}_{\rm RRL} = q_\mu T^{\mu\alpha\beta}_{\rm RLR} = \frac{1}{2}q_\mu T^{\mu\alpha\beta}_{\rm RLL} = -\frac{1}{24\pi^2}\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]}. \tag{2.25b}$$

In contrast, the same correlators read for the case of an anticommuting $\gamma_5^{\rm AC}$

<u>IREG/AC</u>:
$$q_\mu T^{\mu\alpha\beta}_{\rm RRR} = -q_\mu T^{\mu\alpha\beta}_{\rm LLL} = -\frac{1}{12\pi^2}\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]}, \tag{2.26a}$$

$$q_\mu T^{\mu\alpha\beta}_{\rm RRL} = q_\mu T^{\mu\alpha\beta}_{\rm RLR} = q_\mu T^{\mu\alpha\beta}_{\rm RLL} = 0. \tag{2.26b}$$

The crucial difference between these two results is that only Eq. (2.25) are likewise compatible with gauge invariance and Bose symmetry since in this case Bose symmetry does not impose any additional restrictions on the distribution of the anomaly on the pseudo-scalar and the vector current [31]. It is therefore possible to entirely shift the anomaly away from the vector current in order to preserve gauge invariance.

Using the FDF approach, we computed the aforementioned chiral correlators and find agreement with Eq. (2.25), i.e.

<u>FDF</u>:
$$q_\mu T^{\mu\alpha\beta}_{\rm RRR} = -q_\mu T^{\mu\alpha\beta}_{\rm LLL} = -\frac{1}{12\pi^2}\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]} + \mathcal{O}(\epsilon), \tag{2.27a}$$

$$q_\mu T^{\mu\alpha\beta}_{\rm RRL} = q_\mu T^{\mu\alpha\beta}_{\rm RLR} = \frac{1}{2}q_\mu T^{\mu\alpha\beta}_{\rm RLL} = -\frac{1}{24\pi^2}\{\varepsilon^{\alpha\beta\mu\nu}p_{1,\mu}p_{2,\nu}\}_{[4]} + \mathcal{O}(\epsilon). \tag{2.27b}$$

[8] In Ref. [31], the results are parametrized in terms of a parameter a which is related to momentum-routing invariance and therefore to a so-called "surface term" $v_0 \sim (1+a)$. In dimensional regularization, v_0 is set to zero by definition, resulting in $a = -1$.

In FDF, the results are entirely generated by extra-integrals like in Eq. (2.17). Although using a strictly four-dimensional algebra in combination with an anticommuting γ_5, FDF is therefore compatible with Bose symmetry and gauge invariance at the same time. This finding is confirmed by the validity of the vector Ward identities for which we find in FDF

$$\underline{\text{FDF}}:\; p_{1,\alpha} T^{\mu\alpha\beta}_{\text{AVV}} = p_{2,\beta} T^{\mu\alpha\beta}_{\text{AVV}} = 0. \tag{2.28}$$

It should be mentioned explicitly that these findings are a result of the algebraic rules *within* FDF. If we were to evaluate the algebra in the unregularized theory and apply the rules of FDF only afterwards, we would obtain vanishing results for the "mixed" correlators RRL, RLR, RLL like in Eq. (2.26) [although Eqs. (2.24b) and (2.28) would still hold]. Since the analytical expressions are in general divergent, however, it is clear that the application of a proper regularization has to be the initial step that is necessary to avoid ambiguous results.

III. PSEUDOSCALAR FORM FACTORS IN FDH

In the following, we extend the previous findings to the two-loop level by computing the pseudoscalar form factors of quarks and gluons in the FDH scheme. The results of the form factors that are currently available have been obtained by using CDR and γ_5^{BM} as defined in Eq. (2.2), see e.g. [32] and references therein. In the following we consider the form factors up to two loops for

(i) different dimensional schemes, i.e. CDR/HV and FDH, and
(ii) different γ_5 schemes, i.e. γ_5^{BM} and γ_5^{AC}.

In principle, also the FDF scheme is a viable candidate for treating γ_5 in the framework of dimensional regularization. However, since it is (currently) unclear how this approach can be consistently formulated beyond the one-loop level, we do not consider FDF here.

A. Effective Lagrangian

The coupling strength of a pseudo-scalar Higgs boson A to quarks is directly proportional to the respective quark mass. Denoting the pseudoscalar current by $j_{5,k} \equiv i\bar{\psi}_k \gamma_5 \psi_k$, the corresponding Lagrangian can be written as

$$\mathcal{L}_{\text{full}} = \left[\sum_q y_q m_q j_{5,q} + y_t m_t j_{5,t}\right] \frac{A}{v}, \tag{3.1}$$

where v and y_i denote the Higgs vacuum expectation value and dimensionless Yukawa couplings which depend on the underlying theory, respectively, the sum runs over all light quark flavors $q \in \{d, u, s, c, b\}$, and t corresponds to the top quark.

One way to obtain an effective Lagrangian corresponding to Eq. (3.1) is to consider the (all-order) anomalous relation [29] between the pseudoscalar current $j_{5,k}$ and the axial-vector current $j^\mu_{5,k} \equiv \bar{\psi}_k \gamma^\mu \gamma_5 \psi_k$ in the full theory,

$$\partial_\mu \left[\sum_q j^\mu_{5,q} + j^\mu_{5,t}\right] = 2\left[\sum_q m_q j_{5,q} + m_t j_{5,t}\right] + \frac{N_F + 1}{2}\left(\frac{\alpha_s}{4\pi}\right) \varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}, \tag{3.2}$$

where $G^a_{\mu\nu}$ is the gluonic field strength tensor and $\alpha_s = g_s^2/(4\pi)$ denotes the strong coupling. In the limit of a large top mass, $m_t^2 \gg p^2$, the derivative $\partial_\mu j^\mu_{5,t}$ and the masses of the light quarks can be neglected. The (unregularized) effective Lagrangian can then be written as [33]

$$\mathcal{L}_{\text{eff}} = \left[-\frac{\lambda_{\text{G}}}{8}\{\varepsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}\}_{[4]} - \frac{\lambda_{\text{J}}}{2}\left\{\partial_\mu\left(\sum_q \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q\right)\right\}_{[4]}\right] A, \tag{3.3}$$

where the ψ_q are now quark fields in the effective theory. One important feature of the effective Lagrangian is that it does not carry any mass dependence anymore. Although the interaction between a (pseudoscalar) Higgs and quarks vanishes in the full theory if the quark masses are set to zero, in the effective theory we consider the case of N_F massless quarks which are described by the field ψ. The implications of this choice will be discussed below.

In a next step, we study the effective Lagrangian (3.3) in the framework of the aforementioned dimensional schemes. For this, it is useful to envision some universal characteristics of dimensionally regularized quantities. In any dimensional scheme, derivatives and loop momenta are treated as (quasi) d-dimensional objects. In contrast, for the dimensionality of metric tensors, γ matrices, and vector fields there is some freedom which is fixed by the choice of a specific regularization scheme. In CDR, for example, all Lorentz indices (except for the ones of the ε pseudotensor) are treated in d dimensions. The CDR-regularized version of the first curly bracket in Eq. (3.3) therefore reads

$$O_{\text{G,CDR}} \equiv \{\varepsilon^{\mu\nu\rho\sigma}\}_{[4]} \{G^a_{\mu\nu} G^a_{\rho\sigma}\}_{[d]}. \tag{3.4a}$$

The corresponding Feynman rules are given in Appendix A 1.

One key feature of the Feynman rules stemming from operator (3.4a) is that all of them contain (quasi) d-dimensional momenta with uncontracted Lorentz indices. Due to permutations in μ, ν, ρ, σ, the metric tensors in Eqs. (A1c) and (A1d) also have to be considered in d dimensions. The dimensionality of the indices in Eq. (3.4a) is therefore valid in *all* realizations of dimensional regularization, i.e.

$$O_{\mathrm{G,CDR}} = O_{\mathrm{G,HV}} = O_{\mathrm{G,FDH}} = O_{\mathrm{G,DRED}} \equiv O_{\mathrm{G}}. \quad (3.4b)$$

This in particular means that in FDH and DRED no evanescent operators related to ϵ-scalar–Higgs interactions arise at the tree level.

The regularization of the second curly bracket in Eq. (3.3) is more involved due to the treatment of γ_5. According to the discussion in Sec. II we obtain the regularized operators[9]

$$\underline{\mathrm{BM}}\colon\ O^{\mathrm{BM}}_{\mathrm{J,CDR}} = \frac{i}{3!}\{\epsilon^{\mu\nu\rho\sigma}\}_{[4]}\{\partial_\mu(\bar{\psi}\gamma_\nu\gamma_\rho\gamma_\sigma\psi)\}_{[d]} \quad \text{and} \quad (3.5a)$$

$$\underline{\mathrm{AC}}\colon\ O^{\mathrm{AC}}_{\mathrm{J,CDR}} = \{\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5^{\mathrm{AC}}\psi)\}_{[d]}. \quad (3.5b)$$

In analogy to the discussion of operator O_{G} it follows that Eq. (3.5) are valid in *all* implementations of dimensional regularization,

$$O^{\mathrm{GS}}_{\mathrm{J,CDR}} = O^{\mathrm{GS}}_{\mathrm{J,HV}} = O^{\mathrm{GS}}_{\mathrm{J,FDH}} = O^{\mathrm{GS}}_{\mathrm{J,DRED}} \equiv O^{\mathrm{GS}}_{\mathrm{J}}. \quad (3.6)$$

As for operator O_{G}, the corresponding Feynman rules are given in Appendix A 1.

B. Common definition of the form factors

The regularized operators in Eqs. (3.4) and (3.5) give rise to different pseudoscalar form factors of quarks and gluons. So far, in the literature these quantities have been considered in the framework of CDR, using γ_5^{BM} as defined in Eq. (2.2). The quark form factor related to contributions from operator (3.5), for example, is usually defined via squares of the absolute value of the corresponding matrix elements,

$$f^{\mathrm{BM}}_{q,\mathrm{J}} \equiv \sum_{n=0}^{\infty} \frac{\langle M^{\mathrm{BM},(0)}_{q,\mathrm{J}} | M^{\mathrm{BM},(n)}_{q,\mathrm{J}}\rangle}{\langle M^{\mathrm{BM},(0)}_{q,\mathrm{J}} | M^{\mathrm{BM},(0)}_{q,\mathrm{J}}\rangle} \equiv 1 + f^{\mathrm{BM},(1)}_{q,\mathrm{J}} + f^{\mathrm{BM},(2)}_{q,\mathrm{J}} + \mathcal{O}(\alpha_s^3), \quad (3.7)$$

where n denotes the loop order in the perturbative expansion. By definition, each term in the sum contains products of ε pseudotensors. Although the $\varepsilon^{\mu\nu\rho\sigma}$ are strictly four-dimensional objects, in the literature their products are usually treated in d dimensions [35,36],[10]

$$\{E^{\mu_1\mu_2\mu_3\mu_4}E^{\nu_1\nu_2\nu_3\nu_4}\}_{[d]} \equiv \{-g^{\mu_1\nu_1}g^{\mu_2\nu_2}g^{\mu_3\nu_3}g^{\mu_4\nu_4} \pm \text{perm.}\}_{[d]}, \quad (3.8)$$

where "perm." denotes terms originating from further permutations in the Lorentz indices. Even though the application of Eq. (3.8) in general leads to ambiguous results [22], we consider the implications of this choice by using it to evaluate the numerators in Eq. (3.7).

If p_1, p_2 denote the momenta of the external quarks with $q \equiv p_1 + p_2$ and $p_1^2 = p_2^2 \equiv p^2$, we obtain for the first numerator in the perturbative expansion

$$\langle M^{\mathrm{BM},(0)}_{q,\mathrm{J}} | M^{\mathrm{BM},(0)}_{q,\mathrm{J}}\rangle = \left(\frac{i}{3!}\right)^2 q_{\mu_1}q_{\nu_1}\{E^{\mu_1\mu_2\mu_3\mu_4}E^{\nu_1\nu_2\nu_3\nu_4}\}_{[d]}\{\mathrm{Tr}[\gamma_{\mu_4}\gamma_{\mu_3}\gamma_{\mu_2}\not{p}_1\gamma_{\nu_2}\gamma_{\nu_3}\gamma_{\nu_4}\not{p}_2]\}_{[d]}$$
$$= -\frac{1}{3}q^2[q^2(d-4) + p^2(14-2d)](d-3)(d-2). \quad (3.9)$$

It follows that in the massless on-shell case ($p^2 = 0$), the use of Eq. (3.8) serves as an intermediate regularization of the fractions in Eq. (3.7). This regularization has to be introduced since the r.h.s. of Eq. (3.9) vanishes for $d = 4$. Since the regulator drops out in the definition of the form factors, however, the effects of vanishing quark masses are eliminated. In this way, the Lorentz structure related to the pseudoscalar vertex is effectively disentangled from the kinematics of the process and only the (anti)commutation property of the ε pseudotensor is kept.

In contrast, a separation between the Lorentz structure and the mass dependence of the effective Lagrangian is not possible when using an anticommuting γ_5^{AC}. In this case, the square of the absolute value vanishes in the massless on-shell case, even for arbitrary d,

$$\langle M^{\mathrm{AC},(0)}_{q,\mathrm{J}} | M^{\mathrm{AC},(0)}_{q,\mathrm{J}}\rangle \sim \mathrm{Tr}[\gamma_5^{\mathrm{AC}}\not{q}\not{p}_1\not{q}\gamma_5^{\mathrm{AC}}\not{p}_2] = -4q^2p^2. \quad (3.10)$$

An on-shell definition of the form factor for the case of massless quarks similar to the one in Eq. (3.7) is therefore not possible for γ_5^{AC}. The reason is that using Eq. (2.3) as the defining property of γ_5^{AC}, the ε pseudotensor is *implicitly* treated in strictly four dimensions. Like in Eq. (3.9) for $d = 4$, the square of the tree-level amplitudes then vanishes for $p^2 = 0$. In the next section we provide alternative definitions of the pseudoscalar form factors,

[9]Eq. (3.5) is obtained by starting from the unregularized Lagrangian (3.3) and applying the shift of Eq. (2.9) together with Def. (2.2). The structure of operator $O^{\mathrm{BM}}_{\mathrm{J,CDR}}$ has first been discussed in Ref. [34].

[10]In order to distinguish this d-dimensional treatment of the ε pseudotensor from a strictly four-dimensional one we use the symbol E.

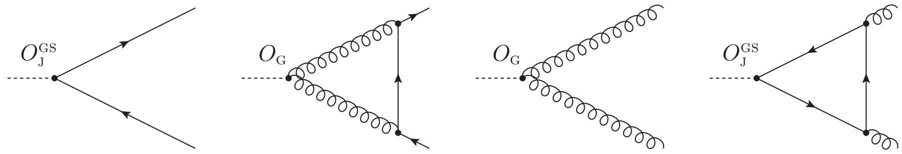

FIG. 3. Lowest-order contributions to the pseudoscalar form factors $\bar{F}_{q,J}^{GS}$, $\bar{F}_{q,G}$, $\bar{F}_{g,G}$, and $\bar{F}_{g,J}^{GS}$ (from left to right). Note, that the "mixed" amplitudes $\bar{M}_{q,G}$ and $\bar{M}_{g,J}$ are loop induced and are therefore at least of $\mathcal{O}(\alpha_s)$.

avoiding the use of Eq. (3.8) and including the case of an anticommuting γ_5^{AC}.

C. Alternative definition and bare results for γ_5^{BM}

Like in Secs. II C and II D, in the following we consider the ε pseudotensor outside dimensional regularization and treat it in strictly four dimensions. In this way it is only the remainder that is dimensionally regularized. Following Ref. [33], we write, for example, the (all-order) contribution of operator O_J^{BM} to the quark form factor as[11]

$$M_{q,J}^{BM} = \{\varepsilon_{\mu\nu\rho\sigma}\}_{[4]}\bar{u}(p_1)\{(R_{q,J}^{BM})^{\mu\nu\rho\sigma}\}_{[d]}v(p_2), \tag{3.11}$$

where u and v denote spinors of the external quarks. By construction, the remainder in the second curly bracket is totally antisymmetric in μ, ν, ρ, σ. Regarding its Lorentz decomposition there is only one structure that is linear in the external momentum q. Making the (anti)symmetrization explicit, we write

$$\begin{aligned}(R_{q,J}^{BM})^{\mu\nu\rho\sigma} &= (q^\mu\gamma^\nu\gamma^\rho\gamma^\sigma - q^\nu\gamma^\mu\gamma^\rho\gamma^\sigma \pm \text{perm.})R_{q,J}^{BM}\\ &\equiv (P_q^{BM})^{\mu\nu\rho\sigma}R_{q,J}^{BM}.\end{aligned} \tag{3.12}$$

For the extraction of the remainder without indices we define the normalization factor

$$\begin{aligned}&\mathrm{Tr}[q_\mu\gamma_\nu\gamma_\rho\gamma_\sigma(P_q^{BM})^{\mu\nu\rho\sigma}]_{[d]}\\ &\quad= \frac{q^2}{6}(d-3)(d-2)(d-1) \equiv N_q^{BM}.\end{aligned} \tag{3.13}$$

The coefficient of the remainder is then obtained by

$$R_{q,J}^{BM} = (N_q^{BM})^{-1}\mathrm{Tr}[q_\mu\gamma_\nu\gamma_\rho\gamma_\sigma(R_{q,J}^{BM})^{\mu\nu\rho\sigma}]. \tag{3.14}$$

In practical calculations we directly implement the quantity $(R_{q,J}^{BM})^{\mu\nu\rho\sigma}$. In other words, we use the Feynman rule in Eq. (A1a) and suppress the ε pseudotensor. Using the projection in Eq. (3.14), this modified Feynman rule is then used to compute one- and two-loop contributions of operator O_J^{BM} to the form factor. In general, these results are UV and IR divergent. After UV renormalization and IR subtraction, however, the limit $d \to 4$ can be taken. A contraction with the four-dimensional indices of $\varepsilon^{\mu\nu\rho\sigma}$ is then possible.

Using this approach and γ_5^{BM} as given in Eq. (2.2), we define the (regularization-scheme dependent) pseudoscalar form factors of quarks

$$\bar{F}_{q,J}^{BM} \equiv \sum_{n=0}^{\infty}\frac{\bar{R}_{q,J}^{BM,(n)}}{R_{q,J}^{BM,(0)}} \equiv 1 + \bar{F}_{q,J}^{BM,(1)} + \bar{F}_{q,J}^{BM,(2)} + \mathcal{O}(\alpha_s^3), \tag{3.15a}$$

$$\bar{F}_{q,G} \equiv \sum_{n=1}^{\infty}\frac{\bar{R}_{q,G}^{(n)}}{R_{q,G}^{(1)}} \equiv 1 + \bar{F}_{q,G}^{(1)} + \mathcal{O}(\alpha_s^2), \tag{3.15b}$$

and gluons

$$\bar{F}_{g,G} \equiv \sum_{n=0}^{\infty}\frac{\bar{R}_{g,G}^{(n)}}{R_{g,G}^{(0)}} \equiv 1 + \bar{F}_{g,G}^{(1)} + \bar{F}_{g,G}^{(2)} + \mathcal{O}(\alpha_s^3), \tag{3.15c}$$

$$\bar{F}_{g,J}^{BM} \equiv \sum_{n=1}^{\infty}\frac{\bar{R}_{g,J}^{BM,(n)}}{R_{g,J}^{BM,(1)}} \equiv 1 + \bar{F}_{g,J}^{BM,(1)} + \mathcal{O}(\alpha_s^2). \tag{3.15d}$$

The notation $\bar{R}_{a,A}^{(n)}$ for the remainders is chosen such that the index n denotes the loop order in the perturbative expansion, $a \in \{q, g\}$ indicates a contribution to the quark or the gluon form factor, and $A \in \{J, G\}$ specifies whether the respective contribution originates from operator O_J or O_G. The explicit definition of the remainders is given in Appendix A 1. To distinguish the underlying regularization we use a bar for quantities in the FDH scheme and no bar for quantities in CDR/HV. Note that contributions related to operator O_J depend on the applied γ_5 scheme which is indicated by the superscript BM.

The lowest-order contributions to the form factors are shown in Fig. 3. As discussed in Sec. III A, they do not depend on the applied version of dimensional regularization, i.e. $\bar{R}_{q,J}^{BM,(0)} = R_{q,J}^{BM,(0)}$, $\bar{R}_{q,G}^{(1)} = R_{q,G}^{(1)}$ and similar for amplitudes with external gluons. At higher perturbative orders, however, FDH results differ from the ones in CDR/HV due to the different treatment of the Lorentz algebra. For the practical calculations in the FDH scheme we follow the guideline given in Sec. 4 of Ref. [26]. More precisely, at the one-loop level we perform the split of Eq. (2.5) and distinguish the evanescent coupling α_e which is related to ϵ-scalar–fermion interactions from the gauge

[11] All other form factors involving ε pseudotensors are treated in the same way. The corresponding definitions are given in Appendix A 1.

coupling α_s.[12] The two-loop calculations are performed by using a (quasi) d_s-dimensional Lorentz algebra as given in Eq. (2.6). Throughout the calculation, d_s is identified with 4.

The one- and two-loop results of the (bare) form factors in FDH are given in Appendix A 2. They have been obtained in the following way: The generation of the diagrams and analytical expressions has been done with the *Mathematica* package FEYNARTS [37]. In order to cope with the Lorentz structure in the FDH scheme we used a modified version of TRACER [38]. The subsequent integral reduction and evaluation has been done with an in-house-algorithm that is based on integration-by-parts identities and the Laporta algorithm [39].

D. Form factors with γ_5^{AC}

As shown in Sec. II C, the evaluation of the Lorentz algebra using an anticommuting γ_5^{AC} may lead to much simpler analytical expressions compared to the case of γ_5^{BM}. Since, for example, one-loop contributions of operator $O_{\mathrm{J}}^{\mathrm{AC}}$ to the quark form factor do not contain traces with γ_5^{AC}, the corresponding amplitude can be written as

$$\bar{M}_{q,\mathrm{J}}^{\mathrm{AC},(1)} = \bar{u}(p_1)\{\not{q}\gamma_5^{\mathrm{AC}}\bar{R}_{q,\mathrm{J}}^{\mathrm{AC},(1)}\}_{[d]}v(p_2). \tag{3.16}$$

Suppressing the spinors and using $(\gamma_5^{\mathrm{AC}})^2 = \mathbb{1}$, the remainder can be extracted via

$$\bar{R}_{q,\mathrm{J}}^{\mathrm{AC},(1)} = \frac{1}{4q^2}\mathrm{Tr}[\gamma_5^{\mathrm{AC}}\not{q}\bar{M}_{q,\mathrm{J}}^{\mathrm{AC},(1)}]_{[d]}. \tag{3.17}$$

This remainder can be used to define a form factor in a similar way as in Eq. (3.15a). As it turns out, however, all perturbative coefficients of the remainder vanish in the massless on-shell case. This can be seen from the explicit analytical expression

$$\bar{M}_{q,\mathrm{J}}^{\mathrm{AC},(1)} \sim \int \frac{d^dk}{(2\pi)^d}\frac{\gamma_{[d_s]}^{\alpha}[(\not{k}+\not{p}_1)\not{q}\gamma_5^{\mathrm{AC}}(\not{k}-\not{p}_2)]_{[d]}\gamma_{[d_s]}^{\beta}(g_{\alpha\beta})_{[d_s]}}{(k+p_1)_{[d]}^2(k-p_2)_{[d]}^2k_{[d]}^2}. \tag{3.18}$$

Anticommuting γ_5^{AC} to the left, the evaluation of the algebra only yields integrals that are scaleless for $p_1^2 = p_2^2 = 0$. Like in Eq. (3.10), a separation between the Lorentz structure and the mass dependence of the effective Lagrangian is then not possible. An anticommuting γ_5^{AC} can therefore not be used to obtain the quark form factor related to Lagrangian (3.1) in a massless framework.[13]

[12]For the definition of α_e we refer to Ref. [24]. The only one-loop diagram $\sim\alpha_e$ that is relevant for the present computation is the right one in Fig. 4.

[13]The fact that the amplitude vanishes for γ_5^{AC} is not a characteristic of the AC scheme itself but of the observable under consideration. Even using γ_5^{BM}, the square of the absolute values in Eq. (3.9) vanishes in the massless on-shell case if the ε pseudotensors are treated in strictly four dimensions.

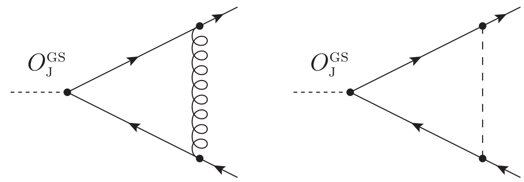

FIG. 4. One-loop diagrams contributing to the form factor $\bar{F}_{q,\mathrm{J}}^{\mathrm{GS}}$ including a gluon (left) and an associated ϵ-scalar (right). The right diagram is proportional to the evanescent coupling α_e and only contributes in the FDH scheme.

However, in Sec. IV B we consider Eq. (3.18) in the massless off-shell case to determine so far unknown UV renormalization constants. In this case, the amplitude has a nonvanishing value.

IV. UV RENORMALIZATION

To obtain UV-renormalized Green functions it is useful to distinguish two classes of contributions,

(i) renormalization of the couplings, fields, and the gauge parameter,

(ii) renormalization of the effective operators O_{G} and $O_{\mathrm{J}}^{\mathrm{GS}}$.

The renormalization of evanescent couplings in the FDH scheme is well known [40,41]. In any l-loop calculation, the coupling α_e describing the interaction of ϵ-scalars and quarks has to be distinguished from the gauge coupling α_s in $(l-1)$-loop contributions [26], see also Fig. 4. The multiplicative coupling renormalization is given by

$$\alpha_i^0 = \left(\frac{\mu_r}{\mu_0}\right)^{2\epsilon}\bar{Z}_{\alpha_i}\alpha_i(\mu_r), \qquad \alpha_i^0 \in \{\alpha_s^0, \alpha_e^0\}, \tag{4.1}$$

where μ_r and μ_0 denote the renormalization scale and the regularization scale, respectively. In the following we set $\mu_r \equiv \mu_0$ and suppress the explicit scale dependence of the renormalized couplings; as renormalization prescription we use the $\overline{\mathrm{MS}}$ scheme. The corresponding renormalization constants in FDH are given in Appendix A 3.

A. Operator renormalization for γ_5^{BM}

To describe the UV behavior of the operators O_{G} and $O_{\mathrm{J}}^{\mathrm{GS}}$, multiplicative renormalization transformations similar to Eq. (4.1) are not sufficient since the operators mix under renormalization. As shown in Sec. III A, the operator basis remains unchanged when using the FDH scheme instead of CDR due to the absence of evanescent operators at the tree-level. The related renormalization constants, however, are different in both schemes. In analogy to the CDR result [6], we therefore write the operator mixing in FDH as[14]

[14]Compared to the original reference we added the superscript GS indicating the dependence on the applied γ_5 scheme. The renormalization constants in CDR are defined in the same way without a bar.

$$\begin{pmatrix} O_{\mathrm{G}} \\ O_{\mathrm{J}} \end{pmatrix}_{\mathrm{ren}} \equiv \begin{pmatrix} \bar{Z}_{\mathrm{GG}} & \bar{Z}_{\mathrm{GJ}} \\ \bar{Z}^{\mathrm{GS}}_{\mathrm{JG}} & \bar{Z}^{\mathrm{GS}}_{\mathrm{JJ}} \end{pmatrix} \begin{pmatrix} O_{\mathrm{G}} \\ O^{\mathrm{GS}}_{\mathrm{J}} \end{pmatrix}_{\mathrm{bare}}. \tag{4.2}$$

The "mixed" constants $\bar{Z}_{\mathrm{GJ}}$ and $\bar{Z}^{\mathrm{GS}}_{\mathrm{JG}}$ are related to UV divergences of the second and the rightmost diagram in Fig. 3, respectively, and to perturbative corrections thereof. As shown in Ref. [42], the latter constant vanishes to all orders in perturbation theory, i. e. $\bar{Z}^{\mathrm{GS}}_{\mathrm{JG}} = 0$. The former, on the other hand, is at least of $\mathcal{O}(\alpha_s)$. Due to the absence of evanescent contributions to the second topology in Fig. 3, its one-loop coefficient is regularization-scheme independent,

$$\bar{Z}^{(1)}_{\mathrm{GJ}} - Z^{(1)}_{\mathrm{GJ}} = 0. \tag{4.3}$$

As discussed in Sec. II C, the use of γ_5^{BM} in a dimensional framework spoils properties of the axial-vector current and the Ward identities. In this case an additional *finite* renormalization $\bar{Z}_5^{\mathrm{BM}}$ has to be introduced to restore the initial properties [43]. We therefore define

$$\bar{Z}^{\mathrm{BM}}_{\mathrm{JJ}} \equiv \bar{Z}^{\mathrm{BM}}_{\overline{\mathrm{MS}}} \bar{Z}^{\mathrm{BM}}_5, \tag{4.4}$$

where $\bar{Z}^{\mathrm{BM}}_{\overline{\mathrm{MS}}}$ only contains pure poles in ϵ for arbitrary n_ϵ. For the operator renormalization in the FDH scheme we then get

$$(O_{\mathrm{G}})_{\mathrm{ren}} = \bar{Z}_{\mathrm{GG}} (O_{\mathrm{G}})_{\mathrm{bare}} + \bar{Z}_{\mathrm{GJ}} (O^{\mathrm{BM}}_{\mathrm{J}})_{\mathrm{bare}}, \tag{4.5a}$$

$$(O_{\mathrm{J}})_{\mathrm{ren}} = \bar{Z}^{\mathrm{BM}}_{\overline{\mathrm{MS}}} \bar{Z}^{\mathrm{BM}}_5 (O^{\mathrm{BM}}_{\mathrm{J}})_{\mathrm{bare}}. \tag{4.5b}$$

The values of $\bar{Z}_{\mathrm{GG}}$, $\bar{Z}_{\mathrm{GJ}}$, and $\bar{Z}^{\mathrm{BM}}_{\overline{\mathrm{MS}}}$ in the FDH scheme can be obtained by making use of the fact that they are the only so far unknown quantities entering the UV-renormalized and IR-subtracted form factors. Using Eq. (4.3) and the structure of the IR divergences given in Eq. (A9), the particular structure of the operator mixing allows one to determine the one- and two-loop renormalization coefficients in a *unique* way.

To illustrate the determination of the renormalization constants we consider the renormalized form factor $\bar{\mathcal{F}}^{\mathrm{BM},(1)}_{q,\mathrm{J}}$ given by Eqs. (A6a) and (A8a) as an example. At the one-loop level, any UV renormalization constant has at most *single* ϵ poles in the framework of dimensional regularization. Depending on which specific scheme is used, the coefficients of these poles differ by terms $\sim n_\epsilon$, depending on the treatment of metric tensors and γ matrices. The scheme-dependent part of a one-loop renormalization constant is therefore finite for $n_\epsilon = 2\epsilon$,

$$(\delta \bar{Z}^{(1)} - \delta Z^{(1)}) = \mathcal{O}(n_\epsilon/\epsilon) = \mathcal{O}(\epsilon/\epsilon) = \mathcal{O}(\epsilon^0). \tag{4.6}$$

In order to make the scheme-dependent terms explicit, however, we leave n_ϵ as an arbitrary variable in the following results. Identifying the (renormalized) couplings, $\alpha_e = \alpha_s$, a comparison of Eq. (A8a) with prediction (A9a) for the IR divergences then yields

$$\bar{\mathcal{F}}^{\mathrm{BM},(1)}_{q,\mathrm{J}} - \mathcal{F}^{\mathrm{BM},(1)}_{q,\mathrm{J}}\Big|_{\mathrm{poles}}$$
$$= \left(\frac{\alpha_s}{4\pi}\right)\left[-C_F \frac{n_\epsilon}{2\epsilon}\right] + \left(\delta \bar{Z}^{\mathrm{BM},(1)}_{\overline{\mathrm{MS}}} - \delta Z^{\mathrm{BM},(1)}_{\overline{\mathrm{MS}}}\right) \tag{4.7a}$$
$$\equiv \bar{\mathbf{Z}}^{(1)}_q - \mathbf{Z}^{(1)}_q = \left(\frac{\alpha_s}{4\pi}\right)\left[+C_F \frac{n_\epsilon}{2\epsilon}\right]. \tag{4.7b}$$

Since $\delta Z^{\mathrm{BM},(1)}_{\overline{\mathrm{MS}}}$ vanishes in CDR [6], $\bar{Z}^{\mathrm{BM}}_{\overline{\mathrm{MS}}}$ receives a non-vanishing one-loop contribution in the FDH scheme which is finite for $n_\epsilon = 2\epsilon$,

$$\delta \bar{Z}^{\mathrm{BM},(1)}_{\overline{\mathrm{MS}}} = \left(\frac{\alpha_s}{4\pi}\right) C_F \frac{n_\epsilon}{\epsilon}. \tag{4.8}$$

All other renormalization coefficients can be obtained in the same way. The explicit calculation yields

$$\begin{aligned}
\bar{Z}^{\mathrm{BM}}_{\overline{\mathrm{MS}}} = 1 &+ \left(\frac{\alpha_s}{4\pi}\right) C_F \frac{n_\epsilon}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \\
&\times \Bigg\{ C_A C_F \left[\frac{22}{3\epsilon} + n_\epsilon\left(-\frac{1}{\epsilon^2} + \frac{11}{3\epsilon}\right) + n_\epsilon^2\left(\frac{1}{2\epsilon^2} + \frac{1}{4\epsilon}\right)\right] \\
&+ C_F^2 \left[n_\epsilon\left(-\frac{1}{\epsilon^2} - \frac{4}{\epsilon}\right) - \frac{3n_\epsilon^2}{4\epsilon}\right] \\
&+ C_F N_F \left[\frac{5}{3\epsilon} + n_\epsilon\left(\frac{1}{2\epsilon^2} - \frac{1}{4\epsilon}\right)\right]\Bigg\} + \mathcal{O}(\alpha_s^3),
\end{aligned} \tag{4.9a}$$

$$\begin{aligned}
\bar{Z}_{\mathrm{GG}} = 1 &+ \left(\frac{\alpha_s}{4\pi}\right)\left\{C_A\left[-\frac{11}{3\epsilon} + \frac{n_\epsilon}{6\epsilon}\right] + N_F \frac{2}{3\epsilon}\right\} \\
&+ \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ C_A^2 \left[\frac{121}{9\epsilon^2} - \frac{17}{3\epsilon} - n_\epsilon\left(\frac{11}{9\epsilon^2} - \frac{7}{6\epsilon}\right) + \frac{n_\epsilon^2}{36\epsilon^2}\right] \\
&+ C_A N_F \left[-\frac{44}{9\epsilon^2} + \frac{5}{3\epsilon} + \frac{2n_\epsilon}{9\epsilon^2}\right] + C_F N_F \left[\frac{1}{\epsilon} - \frac{n_\epsilon}{2\epsilon}\right] \\
&+ N_F^2 \frac{4}{9\epsilon^2}\Bigg\} + \mathcal{O}(\alpha_s^3),
\end{aligned} \tag{4.9b}$$

$$\begin{aligned}
\bar{Z}_{\mathrm{GJ}} = &\left(\frac{\alpha_s}{4\pi}\right) C_F \frac{12}{\epsilon} + \left(\frac{\alpha_s}{4\pi}\right)^2 \\
&\times \Bigg\{ C_A C_F \left[-\frac{44}{\epsilon^2} + \frac{142}{3\epsilon} + n_\epsilon\left(\frac{2}{\epsilon^2} + \frac{2}{3\epsilon}\right)\right] \\
&+ C_F^2 \left[-\frac{42}{\epsilon} + n_\epsilon\left(\frac{6}{\epsilon^2} - \frac{6}{\epsilon}\right)\right] + C_F N_F \left[\frac{8}{\epsilon^2} - \frac{4}{3\epsilon}\right]\Bigg\} \\
&+ \mathcal{O}(\alpha_s^3).
\end{aligned} \tag{4.9c}$$

For $n_\epsilon = 0$, Eq. (4.9) agree with the well-known CDR results given e.g. in Ref. [6]. Like in CDR, $\bar{Z}_{\mathrm{GG}}$ coincides with the renormalization of the gauge coupling, see also Eq. (A7a). The results in Eq. (4.9) have been cross-checked with an explicit calculation of the form factors in the off-shell case, including a renormalization of the external parton fields and the gauge parameter.

The CDR value of the *finite* renormalization constant in Eq. (4.5b) is known up to the two-loop level [6],

$$Z_5^{\rm BM} = 1 + \left(\frac{\alpha_s}{4\pi}\right)\{-4C_F\} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left\{22C_F^2 - \frac{107}{9}C_A + \frac{31}{18}C_F N_F\right\} + \mathcal{O}(\alpha_s^3). \tag{4.10a}$$

In general, UV renormalized and IR subtracted FDH results differ at most by terms of $\mathcal{O}(\epsilon^0 n_\epsilon)$ from the corresponding quantities in CDR. Setting $n_\epsilon = 2\epsilon$ and taking the subsequent limit $\epsilon \to 0$, these differences then vanish. The value of $Z_5^{\rm BM}$ is therefore a regularization-scheme independent quantity to all orders in perturbation theory,

$$\bar{Z}_5^{\rm BM} \equiv Z_5^{\rm BM}. \tag{4.10b}$$

The regularization-scheme dependent renormalization of operator $O_{\rm J}^{\rm BM}$ at the one-loop level has first been studied in Ref. [3].[15] For the finite renormalization, the following results are provided,

$$\delta Z_{\rm finite}^{(1)} = \left(\frac{\alpha_s}{4\pi}\right)\left[-8\frac{C_F}{2}\right], \quad \delta \bar{Z}_{\rm finite}^{(1)} = \left(\frac{\alpha_s}{4\pi}\right)\left[-4\frac{C_F}{2}\right], \tag{4.11}$$

which are valid in CDR (left) and FDH (right). At first sight, there seems to be a contradiction to Eq. (4.10b). However, in Ref. [3] n_ϵ is identified with 2ϵ throughout the calculation. In this way, contributions from $\bar{Z}_{\overline{\rm MS}}^{\rm BM}$ and $Z_5^{\rm BM}$ are combined. The results in Eq. (4.11) are therefore in agreement with a combination of Eqs. (4.9a) and (4.10).

B. Operator renormalization for $\gamma_5^{\rm AC}$

In order to determine the so far unknown renormalization of operator $O_{\rm J}^{\rm AC}$, we consider contributions to $\bar{M}_{q,\rm J}^{\rm AC}$ up to the two-loop level in the off-shell case. Following Ref. [27], it is useful to distinguish two classes of contributions:

(i) Type A: Contributions where the $\gamma_5^{\rm AC}$ vertex is attached to an external quark line, see the left diagram in Fig. 5.
(ii) Type B: Contributions where the $\gamma_5^{\rm AC}$ vertex is attached to a quark loop, see the right diagram in Fig. 5.

1. Type A contributions

Type A contributions to $\bar{M}_{q,\rm J}^{\rm AC}$ can be evaluated in a particular simple way by applying the setup described in Sec. III D. Using $(\gamma_5^{\rm AC})^2 = \mathbb{1}$, all traces can be reduced to expressions without any appearance of $\gamma_5^{\rm AC}$. In this way, no difficulties related to the evaluation of the trace arise. In particular, Type A amplitudes do not contribute to the anomaly. In analogy to the case of $\gamma_5^{\rm BM}$, we therefore write the renormalized operator as

$$\text{Type A}: \ (O_{\rm J})_{\rm ren} = \bar{Z}_{\overline{\rm MS}}^{\rm AC}(O_{\rm J}^{\rm AC})_{\rm bare}. \tag{4.12}$$

As before, $\bar{Z}_{\overline{\rm MS}}^{\rm AC}$ contains pure poles in ϵ for arbitrary n_ϵ. In contrast to Eq. (4.5b), however, we do not include a finite renormalization which is due to the fact that Type A amplitudes are not related to the anomalous contributions to $\bar{M}_{q,\rm J}^{\rm AC}$. The fact that there is no need for the introduction of symmetry-restoring counterterms at the one-loop level when using $\gamma_5^{\rm AC}$ has first been discussed in Ref. [44]. Further evidence for the validity of Eq. (4.12) beyond the one-loop level will be given below.

The so far unknown renormalization constant can be obtained from an off-shell computation of the amplitudes $\bar{M}_{q,\rm J}^{\rm AC}$. We performed the explicit calculation up to the two-loop level and obtain the simple result

$$\bar{Z}_{\overline{\rm MS}}^{\rm AC} = 1 + \mathcal{O}(\alpha_s^3). \tag{4.13}$$

The renormalization of operator $O_{\rm J}^{\rm AC}$ is therefore trivial, at least up to two loops. This result is closely related to the use of an anticommutator in the right definition of Eq. (2.8a). If we were to define $[\gamma_5^{\rm AC}, \gamma_{[n_\epsilon]}^\mu] \equiv 0$ instead, $\delta\bar{Z}_{\overline{\rm MS}}^{\rm AC}$ would have a nonvanishing value starting at one loop. In the same way it is the different treatment of strictly 4- and $(d-4)$-dimensional quantities in Eq. (2.4) that results in the nonvanishing constant $\delta\bar{Z}_{\overline{\rm MS}}^{\rm BM}$ given in Eq. (4.9a).

2. Type B contributions

Type B contributions include traces like in Eq. (2.20). Let us first consider the anomalous quark loops shown in

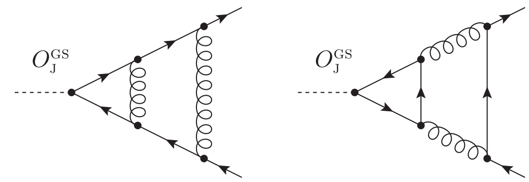

FIG. 5. Sample diagrams contributing to the form factor $\bar{F}_{q,\rm J}^{\rm GS}$ at the two-loop level.

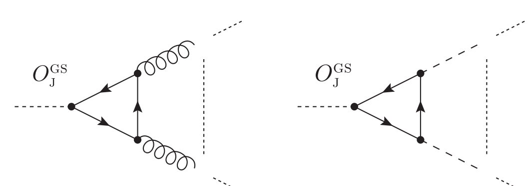

FIG. 6. Anomalous (sub)diagrams related to operator $O_{\rm J}^{\rm GS}$ with gluons (left) and ϵ-scalars (right) attached to the loop. The left diagram is only present in FDH and vanishes according to its Lorentz structure.

[15]In this reference, the underlying regularization is called "supersymmetric dimensional regularization" (SDR) which in our nomenclature corresponds to dimensional reduction (DRED). Since, however, contributions with external ϵ-scalars vanish for the pseudoscalar form factors, the results coincide with the ones in FDH.

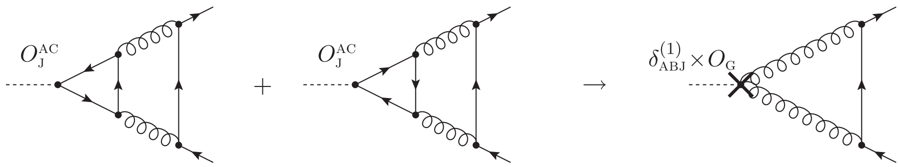

FIG. 7. Equivalence between anomalous two-loop contributions to $\bar{M}_{q,\mathrm{J}}^{\mathrm{AC}}$ and $\delta_{\mathrm{ABJ}}^{(1)}\bar{M}_{q,\mathrm{G}}^{(1)}$.

Fig. 6. These diagrams either yield direct contributions to the gluon form factor at the one-loop level or they contribute as subdiagrams at higher loop orders. Their one-loop result has been obtained in Sec. II D by using γ_5^{BM} and the FDF framework, respectively. Generalizing to the case of QCD, we write the corresponding amplitude as

$$(M_{g,\mathrm{J}}^{\mathrm{BM},(1)})_{\alpha\beta}^{ab} = i\left(\frac{\alpha_s}{4\pi}\right)N_F T_F \delta^{ab}\{\epsilon_{\alpha\beta\mu\nu}\}_{[4]}\{l_1{}^{\mu} l_2{}^{\nu}\}_{[d]} + \mathcal{O}(\epsilon), \tag{4.14a}$$

$$\equiv i\delta_{\mathrm{ABJ}}^{(1)}(\alpha_s)\delta^{ab}\{\epsilon_{\alpha\beta\mu\nu}\}_{[4]}\{l_1{}^{\mu} l_2{}^{\nu}\}_{[d]} + \mathcal{O}(\epsilon), \tag{4.14b}$$

where the l_1, l_2 are line momenta attached to the loop. Since momenta do not contain evanescent degrees of freedom it follows that quark loops with external ϵ-scalars vanish. The fact that the result in Eq. (4.14) is regularization-scheme independent has first been found in Ref. [3].

To obtain a similar result with γ_5^{AC} it is in principle necessary to modify the trace operation. These redefinitions, however, are usually made in such a way that they reproduce Eq. (4.14). Instead of rederiving the already known result in a different framework we directly use it in practical computations. This is done by realizing that the Lorentz and the color structure in Eq. (4.14) are exactly the same as in the Feynman rule given in Eq. (A1c). Accordingly, for Type B contributions the renormalization of operator O_{J} is closely related to the one of O_{G}, see Fig. 7. Up to the two-loop level we therefore write

$$\text{Type B}: (O_{\mathrm{J}}^{\mathrm{AC}})_{\mathrm{ren}} \equiv \delta_{\mathrm{ABJ}}^{(1)}(\alpha_s) \times (O_{\mathrm{G}})_{\mathrm{ren}} + \mathcal{O}(\alpha_s^3). \tag{4.15}$$

In this way, γ_5 is effectively removed from the computation. The necessary one-loop renormalization of operator O_{G} does not depend on the treatment of γ_5 and is known from Sec. IV A.[16]

[16]The approach of evaluating Type A contributions using γ_5^{AC} and Type B contributions using γ_5^{BM} has been discussed before in Ref. [45]. In this reference, however, the right diagram in Fig. 7 is evaluated as a whole by using projections that lead to similar expressions as in Eq. (3.8). Accordingly, the ε pseudotensor is treated in $d \neq 4$ dimensions and additional finite counterterms have to be added to obtain the correct result. In Eq. (4.15), on the other hand, the known $\mathcal{O}(\alpha_s)$ value of the anomaly is used to effectively reduce the evaluation of the two-loop diagram to a one-loop problem that does not depend on the specific treatment of γ_5.

3. Comparison of BM and AC

With the results of the previous sections it is possible to compare the UV-renormalized off-shell values of $\bar{\mathcal{F}}_{q,\mathrm{J}}^{\mathrm{GS}}$ obtained in BM and AC,

$$\bar{\mathcal{F}}_{q,\mathrm{J}}^{\mathrm{BM}} = Z_5^{\mathrm{BM}} \bar{Z}_{\overline{\mathrm{MS}}}^{\mathrm{BM}} (\bar{F}_{q,\mathrm{J}}^{\mathrm{BM}})_{\mathrm{ren}} + \mathcal{O}(\alpha_s^3), \tag{4.16a}$$

$$\bar{\mathcal{F}}_{q,\mathrm{J}}^{\mathrm{AC}} = \underbrace{(\bar{F}_{q,\mathrm{J}}^{\mathrm{AC}})_{\mathrm{ren}}}_{\text{Type A}} + \underbrace{\delta_{\mathrm{ABJ}}^{(1)}[\bar{R}_{q,\mathrm{G}}^{(1)}/R_{q,\mathrm{J}}^{(0),\mathrm{AC}} + \delta\bar{Z}_{\mathrm{GJ}}^{(1)}]}_{\text{Type B}} + \mathcal{O}(\alpha_s^3). \tag{4.16b}$$

The subscript "ren" indicates that a coupling, gauge parameter, and field (sub)renormalization is applied to the bare coefficients. Taking the limit $\epsilon \to 0$, we find that both results in Eq. (4.16) coincide,

$$\text{offshell}: \bar{\mathcal{F}}_{q,\mathrm{J}}^{\mathrm{BM}}|_{\epsilon\to 0} \equiv \bar{\mathcal{F}}_{q,\mathrm{J}}^{\mathrm{AC}}|_{\epsilon\to 0} + \mathcal{O}(\alpha_s^3). \tag{4.17}$$

This provides further evidence for the fact that there is no need for the introduction of finite counterterms when using γ_5^{AC}. Compared to BM, therefore not only the evaluation of the algebra is much simpler but also the renormalization of operator $O_{\mathrm{J}}^{\mathrm{AC}}$.

Extending these considerations to higher loop orders, it is possible to determine the so far unknown three-loop value of Z_5^{BM} from a genuine three-loop calculation. So far, the standard way to obtain Z_5^{BM} is to consider the (anomalous) relation between the axial-vector and the pseudo-scalar current in the effective theory for the case of N_F massless quarks and to evaluate it between two gluon states (see e.g. Ref. [6]). Since the anomaly itself is of $\mathcal{O}(\alpha_s)$, however, the l-loop coefficient of Z_5^{BM} has to be obtained from an $(l+1)$-loop calculation. In contrast, using an extension of Eqs. (4.16) and (4.17) beyond the two-loop level allows one to determine the same coefficient from an l-loop calculation.

C. UV renormalized form factors

Using the results of the renormalization constants from the previous sections together with Eq. (A8), the UV renormalized form factors in the FDH scheme finally read

$$\begin{aligned}\bar{\mathcal{F}}^{\rm BM}_{q,\rm J} = 1 &+ \left(\frac{\alpha_s}{4\pi}\right)\left\{C_F\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-5+\frac{\pi^2}{6}+\epsilon\left(3+\frac{\pi^2}{4}+\frac{14}{3}\zeta(3)\right)-\epsilon^2\left(3-\frac{\pi^2}{4}-7\zeta(3)-\frac{47}{720}\pi^4\right)\right]+\mathcal{O}(\epsilon^3)\right\}\\
&+\left(\frac{\alpha_s}{4\pi}\right)^2\left\{C_AC_F\left[\frac{11}{2\epsilon^3}+\frac{\frac{23}{18}+\frac{\pi^2}{6}}{\epsilon^2}-\frac{\frac{1075}{108}+\frac{11}{12}\pi^2-13\zeta(3)}{\epsilon}-\frac{25279}{648}-\frac{46}{27}\pi^2+\frac{313}{9}\zeta(3)+\frac{11}{45}\pi^4\right]\right.\\
&+C_F^2\left[\frac{2}{\epsilon^4}+\frac{6}{\epsilon^3}+\frac{\frac{29}{2}-\frac{\pi^2}{3}}{\epsilon^2}+\frac{\frac{77}{4}-\frac{64}{3}\zeta(3)}{\epsilon}+\frac{139}{8}-\frac{\pi^2}{4}-58\zeta(3)-\frac{13}{36}\pi^4\right]\\
&\left.+C_FN_F\left[-\frac{1}{\epsilon^3}-\frac{4}{9\epsilon^2}+\frac{\frac{46}{27}+\frac{\pi^2}{6}}{\epsilon}-\frac{1679}{162}+\frac{23}{54}\pi^2+\frac{2}{9}\zeta(3)\right]+\mathcal{O}(\epsilon)\right\}+\mathcal{O}(\alpha_s^3),\end{aligned} \tag{4.18a}$$

$$\begin{aligned}\bar{\mathcal{F}}_{q,\rm G} = &\left(\frac{\alpha_s}{4\pi}\right)\left\{C_A\left[\frac{7115}{324}-\frac{\pi^2}{9}-2\zeta(3)+\epsilon\left(\frac{111\,049}{1944}-\frac{7321}{11\,664}\pi^2-8\zeta(3)-\frac{53}{1620}\pi^4-\frac{\pi^2}{18}\zeta(3)\right)\right.\right.\\
&\left.+\epsilon^2\left(\frac{660\,451}{3888}-\frac{17\,335}{7776}\pi^2-\frac{80\,515}{2916}\zeta(3)-\frac{300\,449}{2\,099\,520}\pi^4-20\zeta(5)-\frac{53}{58\,320}\pi^6-\frac{19}{81}\pi^2\zeta(3)-\frac{14}{9}\zeta(3)^2-\frac{\pi^4}{648}\zeta(3)\right)\right]\\
&+C_F\left[-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-\frac{29}{4}+\frac{\pi^2}{6}+\epsilon\left(-\frac{203}{24}-\frac{\pi^2}{16}+\frac{14}{3}\zeta(3)\right)+\epsilon^2\left(-\frac{1115}{144}-\frac{947}{864}\pi^2+\frac{127}{12}\zeta(3)+\frac{163}{2880}\pi^4\right)\right]\\
&+N_F\left[-\frac{445}{162}+\epsilon\left(-\frac{8231}{972}+\frac{239}{5832}\pi^2+\frac{4}{3}\zeta(3)\right)\right.\\
&\left.\left.+\epsilon^2\left(-\frac{50\,533}{1944}+\frac{1835}{11\,664}\pi^2+\frac{9125}{1458}\zeta(3)+\frac{22\,903}{1\,049\,760}\pi^4+\frac{1}{27}\pi^2\zeta(3)\right)\right]+\mathcal{O}(\epsilon^3)\right\}+\mathcal{O}(\alpha_s^2),\end{aligned} \tag{4.18b}$$

$$\begin{aligned}\bar{\mathcal{F}}_{g,\rm G} = 1 &+ \left(\frac{\alpha_s}{4\pi}\right)\left\{C_A\left[-\frac{2}{\epsilon^2}-\frac{11}{3\epsilon}+\frac{13}{3}+\frac{\pi^2}{6}+\epsilon\left(12+\frac{14}{3}\zeta(3)\right)+\epsilon^2\left(28-\frac{\pi^2}{3}+\frac{47\pi^4}{720}\right)\right]+\frac{2N_F}{3\epsilon}+\mathcal{O}(\epsilon^3)\right\}\\
&+\left(\frac{\alpha_s}{4\pi}\right)^2\left\{C_A^2\left[\frac{2}{\epsilon^4}+\frac{77}{6\epsilon^3}+\frac{\frac{5}{9}-\frac{\pi^2}{6}}{\epsilon^2}-\frac{\frac{1444}{27}+\frac{11}{36}\pi^2+\frac{25}{3}\zeta(3)}{\epsilon}-\frac{2882}{81}+\frac{29}{9}\pi^2-33\zeta(3)-\frac{7}{60}\pi^4\right]\right.\\
&+C_AN_F\left[-\frac{7}{3\epsilon^3}-\frac{13}{3\epsilon^2}+\frac{\frac{148}{27}+\frac{\pi^2}{18}}{\epsilon}-\frac{295}{81}-\frac{5}{18}\pi^2-2\zeta(3)\right]\\
&\left.+C_FN_F\left[\frac{1}{\epsilon}-\frac{74}{3}+8\zeta(3)\right]+N_F^2\frac{4}{9\epsilon^2}+\mathcal{O}(\epsilon)\right\}+\mathcal{O}(\alpha_s^3),\end{aligned} \tag{4.18c}$$

$$\begin{aligned}\bar{\mathcal{F}}^{\rm BM}_{g,\rm J} = &\left(\frac{\alpha_s}{4\pi}\right)\left\{C_A\left[-\frac{2}{\epsilon^2}-\frac{11}{3\epsilon}+\frac{13}{3}+\frac{\pi^2}{6}+\epsilon\left(16-\frac{\pi^2}{3}+\frac{32}{3}\zeta(3)\right)+\epsilon^2\left(\frac{152}{3}-\frac{4}{3}\pi^2+2\zeta(3)+\frac{127}{720}\pi^4\right)\right]\right.\\
&\left.+C_F\left[\epsilon(10-12\zeta(3))+\epsilon^2\left(38-\frac{7}{6}\pi^2-18\zeta(3)-\frac{\pi^4}{5}\right)\right]+\frac{2N_F}{3\epsilon}+\mathcal{O}(\epsilon^3)\right\}+\mathcal{O}(\alpha_s^3).\end{aligned} \tag{4.18d}$$

Compared to the CDR results which are given e.g. in Ref. [32], the one-loop coefficients differ by terms of $\mathcal{O}(\epsilon^0)$, whereas at the two-loop level these differences are of $\mathcal{O}(\epsilon^{-2})$. After subtracting the IR divergences and taking the physical limit $\epsilon \to 0$, however, we obtain the same (regularization-scheme independent) results.

V. CONCLUSIONS

In this article we discussed the regularization-scheme dependent treatment of γ_5 within dimensional regularization. So far, CDR in combination with $\gamma_5^{\rm BM}$ as defined in Eq. (2.2) has been the most commonly used approach to perform perturbative computations in the dimensional framework. One main reason might be that the approach is based on an explicit construction prescription which enables the use of standard calculational techniques like cyclicity of the trace. At the practical level, however, the evaluation of the algebra is cumbersome due to the increased number of γ matrices and the *ad hoc* (anti) symmetrization of $\gamma_5^{\rm BM}$ operators. Moreover, since initial symmetries are broken explicitly there is an immanent need for the introduction of additional counterterms to obtain correct results. In comparison, the application of an anticommuting $\gamma_5^{\rm AC}$ simplifies the evaluation of the Lorentz algebra significantly which is due to the fact that algebraic properties remain unchanged compared to the unregularized theory. This, however, is not the case for $\gamma_5^{\rm AC}$-odd traces. Since these either vanish or do not exhibit cyclicity, special attention has to be paid to the nonbreaking of gauge invariance and other symmetries of the underlying theory.

At the one-loop level, the FDF approach avoids all these complications related to the treatment of γ_5 since, using a strictly four-dimensional algebra, the matrices $\gamma_5^{\rm BM}$ and $\gamma_5^{\rm AC}$ as well as their algebraic behavior are identical. In Secs. II C and II D, FDF has proven as an effective implementation of the Lorentz algebra that reduces the technical complexity significantly, even including contributions to the axial anomaly. At the same time the results are compatible with gauge invariance and Bose symmetry. In the examples considered, the FDF results are entirely given by so-called extra integrals which can be evaluated in a particular simple way. The question whether FDF can be extended beyond the one-loop level, such that it leads to a facilitation compared to more traditional schemes remains to be answered.

At the two-loop level, we investigated the possibility of utilizing the benefits of different γ_5 schemes and computed the pseudo-scalar form factors of quarks and gluons in the FDH scheme. We have shown explicitly that evanescent Higgs–ϵ-scalar interactions are absent and determined the so far unknown UV renormalization of the corresponding operators. The results of the UV-renormalized form factors are compatible with the general prediction for IR divergences in FDH. As a general recommendation for the treatment of γ_5 we find that the use of $\gamma_5^{\rm BM}$ should be avoided whenever $\gamma_5^{\rm AC}$ leads to an obvious and immediate simplification. This clearly applies to Type A contributions to the pseudo-scalar form factors where not only the evaluation of analytical expressions is simplified but also the operator renormalization when using $\gamma_5^{\rm AC}$. It should be mentioned explicitly that these simplifications are not restricted to FDH but apply to all considered dimensional schemes. For the evaluation of (anomalous) γ_5-odd expressions (like Type B contributions in Sec. IV B), however, it is not clear at all, if the use of $\gamma_5^{\rm AC}$ leads to a perceptible simplification due to the aforementioned complications. In this case, the use of $\gamma_5^{\rm BM}$ therefore still constitutes a viable alternative. Moreover, seizing a suggestion of Ref. [9], Type B contributions can be obtained by removing γ_5 in analytical expressions altogether. This can be done by using the well-known and scheme-independent results of the anomalies. In this way not only the evaluation of the amplitudes is significantly simplified but also the related operator renormalization.

Finally it should be mentioned that observables related to Lagrangian (3.1) are usually obtained in an effective theory for the case of massless quarks. One requirement for this option is that the Lorentz structure related to the γ_5 vertex can be effectively disentangled from the kinematics of the underlying process. As it turned out, for the schemes considered in this article this is only possible for $\gamma_5^{\rm BM}$. For the choice of a particular γ_5 scheme one therefore has to compare the complexity of a calculation with massless quarks and the extended $\gamma_5^{\rm BM}$ algebra with the complexity of a calculation with massive quarks and a simplified $\gamma_5^{\rm AC}$ algebra. The decision which of these alternatives is the more efficient one remains to be made on an individual basis.

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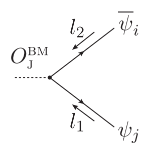
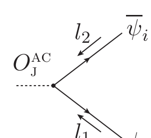
APPENDIX Calculational details

1. Feynman rules and definition of the form factors

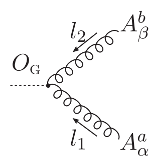
The Feynman rules originating from the effective Lagrangian (3.3) read



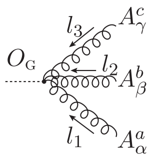
$$= \frac{-i}{4!\,3!\,2}\,\delta_{ij}\left\{\epsilon_{\mu\nu\rho\sigma}\right\}_{[4]}\left\{(l_1+l_2)^\mu\,\gamma^\nu\gamma^\rho\gamma^\sigma \pm \text{perm.}\right\}_{[d]}, \tag{A1a}$$



$$= -i\,\delta_{ij}\,(\not{l}_1+\not{l}_2)\,\gamma_5^{\rm AC}, \tag{A1b}$$



$$= -\frac{i}{4!}\,\delta^{ab}\left\{\epsilon_{\mu\nu\rho\sigma}\right\}_{[4]}\left\{l_1^\mu\,l_2^\nu\,g^\rho{}_\alpha\,g^\sigma{}_\beta \pm \text{perm.}\right\}_{[d]}, \tag{A1c}$$



$$= -\frac{i}{4!}\, f^{abc} \Big\{\epsilon_{\mu\nu\rho\sigma}\Big\}_{[4]} \Big\{(l_1+l_2+l_3)^\mu\, g^\nu{}_\alpha\, g^\rho{}_\beta\, g^\sigma{}_\gamma \pm \text{perm.}\Big\}_{[d]}, \tag{A1d}$$

where "perm" denotes terms originating from further permutations in the indices μ, ν, ρ, σ.

According to the discussion in Sec. III C, we decompose pseudoscalar amplitudes as

$$\bar{M}^{\mathrm{BM}}_{q,\mathrm{J}} = \{\epsilon_{\mu\nu\rho\sigma}\}_{[4]}\bar{u}(p_1)\sum_{n=0}\{(\bar{R}^{\mathrm{BM},(n)}_{q,\mathrm{J}})^{\mu\nu\rho\sigma}\}_{[d]}v(p_2), \tag{A2a}$$

$$\bar{M}_{q,\mathrm{G}} = \{\epsilon_{\mu\nu\rho\sigma}\}_{[4]}\bar{u}(p_1)\sum_{n=1}\{(\bar{R}^{(n)}_{q,\mathrm{G}})^{\mu\nu\rho\sigma}\}_{[d]}v(p_2), \tag{A2b}$$

$$\bar{M}_{g,\mathrm{G}} = \{\epsilon_{\mu\nu\rho\sigma}\}_{[4]}\sum_{n=0}\{(\bar{R}^{(n)}_{g,\mathrm{G}})^{\mu\nu\rho\sigma}_{\alpha\beta}\}_{[d]}\epsilon^\alpha(p_1)\epsilon^\beta(p_2), \tag{A2c}$$

$$\bar{M}^{\mathrm{BM}}_{g,\mathrm{J}} = \{\epsilon_{\mu\nu\rho\sigma}\}_{[4]}\sum_{n=1}\{(\bar{R}^{\mathrm{BM},(n)}_{g,\mathrm{J}})^{\mu\nu\rho\sigma}_{\alpha\beta}\}_{[d]}\epsilon^\alpha(p_1)\epsilon^\beta(p_2), \tag{A2d}$$

where v, u are (anti)quark spinors and ϵ^μ are polarization vectors of the gluon. The sum of the (outgoing) momenta p_1 and p_2 is given by $p_1+p_2=q$. According their Lorentz decomposition, the remainders can be written as

$$(\bar{R}^{\mathrm{BM},(n)}_{q,\mathrm{J}})^{\mu\nu\rho\sigma} \equiv \bar{R}^{\mathrm{BM},(n)}_{q,\mathrm{J}}\{q^\mu\gamma^\nu\gamma^\rho\gamma^\sigma \pm \text{perm}\} \equiv \bar{R}^{\mathrm{BM},(n)}_{q,\mathrm{J}}(P_q)^{\mu\nu\rho\sigma}, \tag{A3a}$$

$$(\bar{R}^{(n)}_{q,\mathrm{G}})^{\mu\nu\rho\sigma} \equiv \bar{R}^{(n)}_{q,\mathrm{G}}(P_q)^{\mu\nu\rho\sigma}, \tag{A3b}$$

$$(\bar{R}^{(n)}_{g,\mathrm{G}})^{\mu\nu\rho\sigma}_{\alpha\beta} \equiv \bar{R}^{(n)}_{g,\mathrm{G}}\{p_1^\mu p_2^\nu g^\rho_\alpha g^\sigma_\beta \pm \text{perm}\} \equiv \bar{R}^{(n)}_{g,\mathrm{G}}(P_g)^{\mu\nu\rho\sigma}_{\alpha\beta}, \tag{A3c}$$

$$(\bar{R}^{\mathrm{BM},(n)}_{g,\mathrm{J}})^{\mu\nu\rho\sigma}_{\alpha\beta} \equiv \bar{R}^{\mathrm{BM},(n)}_{g,\mathrm{J}}(P_g)^{\mu\nu\rho\sigma}_{\alpha\beta}, \tag{A3d}$$

For the extraction of the coefficients on the r.h.s. of Eq. (A3) we define the following normalization factors,

$$\mathrm{Tr}[q_\mu\gamma_\nu\gamma_\rho\gamma_\sigma(P_q)^{\mu\nu\rho\sigma}] = \frac{1}{6}(d-1)(d-2)(d-3)q^2 \equiv N_q, \tag{A4a}$$

$$(P_g)^2 = -\frac{1}{144}(d-2)(d-3)q^4 \equiv N_g. \tag{A4b}$$

The remainders entering Eq. (3.15) are then obtained by

$$\bar{R}^{\mathrm{BM},(n)}_{q,\mathrm{J}} = (N_q)^{-1}\mathrm{Tr}[q_\mu\gamma_\nu\gamma_\rho\gamma_\sigma(\bar{R}^{\mathrm{BM},(n)}_{q,\mathrm{J}})^{\mu\nu\rho\sigma}], \tag{A5a}$$

$$\bar{R}^{(n)}_{q,\mathrm{G}} = (N_q)^{-1}\mathrm{Tr}[q_\mu\gamma_\nu\gamma_\rho\gamma_\sigma(\bar{R}^{(n)}_{q,\mathrm{G}})^{\mu\nu\rho\sigma}], \tag{A5b}$$

$$\bar{R}^{(n)}_{g,\mathrm{G}} = (N_g)^{-1}(P_g)^{\alpha\beta}_{\mu\nu\rho\sigma}(\bar{R}^{(n)}_{g,\mathrm{G}})^{\mu\nu\rho\sigma}_{\alpha\beta}, \tag{A5c}$$

$$\bar{R}^{\mathrm{BM},(n)}_{g,\mathrm{J}} = (N_g)^{-1}(P_g)^{\alpha\beta}_{\mu\nu\rho\sigma}(\bar{R}^{\mathrm{BM},(n)}_{g,\mathrm{J}})^{\mu\nu\rho\sigma}_{\alpha\beta}. \tag{A5d}$$

2. Bare on-shell results

The nonvanishing coefficients of the bare form factors defined in Eq. (3.15) read

$$\bar{F}_{q,\mathrm{J}}^{\mathrm{BM},(1)} = \left(\frac{\alpha_s}{4\pi}\right) C_F \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 2 + \frac{\pi^2}{6} + \epsilon\left(2 + \frac{\pi^2}{4} + \frac{14}{3}\zeta(3)\right) + \epsilon^2\left(10 + \frac{\pi^2}{6} + 7\zeta(3) + \frac{47}{720}\pi^4\right)\right] + \left(\frac{\alpha_e}{4\pi}\right) C_F \left[-1 - 5\epsilon + \epsilon^2\left(-13 + \frac{\pi^2}{12}\right)\right] + \mathcal{O}(\epsilon^3), \tag{A6a}$$

$$\begin{aligned}\bar{F}_{q,\mathrm{J}}^{\mathrm{BM},(2)} = \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ & C_A C_F \left[-\frac{11}{6\epsilon^3} - \frac{\frac{163}{18} - \frac{\pi^2}{6}}{\epsilon^2} - \frac{\frac{2551}{108} + \frac{11}{36}\pi^2 - 13\zeta(3)}{\epsilon} - \frac{23\,623}{648} - \frac{91}{108}\pi^2 + \frac{467}{9}\zeta(3) + \frac{11}{45}\pi^4\right] \\ & + C_F^2 \left[\frac{2}{\epsilon^4} + \frac{6}{\epsilon^3} + \frac{\frac{21}{2} - \frac{\pi^2}{3}}{\epsilon^2} + \frac{\frac{53}{4} - \frac{64}{3}\zeta(3)}{\epsilon} - \frac{53}{8} + \frac{\pi^2}{12} - 58\zeta(3) - \frac{13}{36}\pi^4\right] \\ & + C_F N_F \left[\frac{1}{3\epsilon^3} + \frac{14}{9\epsilon^2} + \frac{\frac{37}{27} + \frac{\pi^2}{18}}{\epsilon} - \frac{1283}{162} + \frac{7}{27}\pi^2 - \frac{26}{9}\zeta(3)\right]\Bigg\} + \mathcal{O}(\epsilon^1),\end{aligned} \tag{A6b}$$

$$\begin{aligned}\bar{F}_{q,\mathrm{G}}^{(1)} = \left(\frac{\alpha_s}{4\pi}\right) \Bigg\{ & C_A \left[\frac{11}{3\epsilon} + \frac{263}{18} + \epsilon\left(\frac{4949}{108} - \frac{23}{36}\pi^2 - 6\zeta(3)\right) + \epsilon^2\left(\frac{87\,917}{648} - \frac{479}{216}\pi^2 - \frac{257}{9}\zeta(3) - \frac{4}{45}\pi^4\right)\right] \\ & + C_F \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - \frac{11}{2} + \frac{\pi^2}{6} + \epsilon\left(-\frac{37}{4} + \frac{\pi^2}{4} + \frac{14}{3}\zeta(3)\right) + \epsilon^2\left(-\frac{103}{8} - \frac{13}{24}\pi^2 + 7\zeta(3) + \frac{47}{720}\pi^4\right)\right] \\ & + N_F \left[-\frac{2}{3\epsilon} - \frac{19}{9} - \epsilon\left(\frac{355}{54} - \frac{\pi^2}{18}\right) - \epsilon^2\left(\frac{6523}{324} - \frac{19}{108}\pi^2 - \frac{50}{9}\zeta(3)\right)\right]\Bigg\} + \mathcal{O}(\epsilon^3),\end{aligned} \tag{A6c}$$

$$\bar{F}_{g,\mathrm{G}}^{(1)} = \left(\frac{\alpha_s}{4\pi}\right) C_A \left[-\frac{2}{\epsilon^2} + 4 + \frac{\pi^2}{6} + \epsilon\left(12 + \frac{14}{3}\zeta(3)\right) + \epsilon^2\left(28 - \frac{\pi^2}{3} + \frac{47}{720}\pi^4\right)\right] + \mathcal{O}(\epsilon^3), \tag{A6d}$$

$$\begin{aligned}\bar{F}_{g,\mathrm{G}}^{(2)} = \left(\frac{\alpha_s}{4\pi}\right)^2 \Bigg\{ & C_A^2 \left[\frac{2}{\epsilon^4} - \frac{11}{6\epsilon^3} - \frac{\frac{104}{9} + \frac{\pi^2}{6}}{\epsilon^2} - \frac{\frac{433}{27} - \frac{11}{12}\pi^2 + \frac{25}{3}\zeta(3)}{\epsilon} + \frac{3832}{81} + \frac{28\pi^2}{9} + \frac{11}{9}\zeta(3) - \frac{7\pi^4}{60}\right] \\ & + C_A N_F \left[\frac{1}{3\epsilon^3} + \frac{5}{9\epsilon^2} - \frac{\frac{53}{27} + \frac{\pi^2}{6}}{\epsilon} - \frac{1591}{81} - \frac{5\pi^2}{18} - \frac{74}{9}\zeta(3)\right] + C_F N_F \left[-\frac{6}{\epsilon} - \frac{125}{3} + 8\zeta(3)\right]\Bigg\} + \mathcal{O}(\epsilon^1),\end{aligned} \tag{A6e}$$

$$\begin{aligned}\bar{F}_{g,\mathrm{J}}^{\mathrm{BM},(1)} = \left(\frac{\alpha_s}{4\pi}\right) \Bigg\{ & C_A \left[-\frac{2}{\epsilon^2} + 4 + \frac{\pi^2}{6} + \epsilon\left(16 - \frac{\pi^2}{3} + \frac{32}{3}\zeta(3)\right) + \epsilon^2\left(\frac{152}{3} - \frac{4}{3}\pi^2 + 2\zeta(3) + \frac{127}{720}\pi^4\right)\right] \\ & + C_F \left[2 + \epsilon(10 - 12\zeta(3)) + \epsilon^2\left(38 - \frac{7}{6}\pi^2 - 18\zeta(3) - \frac{\pi^4}{5}\right)\right]\Bigg\} + \mathcal{O}(\epsilon^3).\end{aligned} \tag{A6f}$$

3. UV renormalization

The UV renormalization of the couplings α_s and α_e is given by [24]

$$\bar{Z}_{\alpha_s} = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left\{-\frac{\bar{\beta}_{20}^s}{\epsilon}\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{(\bar{\beta}_{20}^s)^2}{\epsilon^2} - \frac{\bar{\beta}_{30}^s + \bar{\beta}_{21}^s}{2\epsilon}\right\} + \mathcal{O}(\alpha_s^3), \tag{A7a}$$

$$\bar{Z}_{\alpha_e} = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left\{-\frac{\bar{\beta}_{11}^e + \bar{\beta}_{02}^e}{\epsilon}\right\} + \mathcal{O}(\alpha_s^2), \tag{A7b}$$

including the β coefficients

$$\bar{\beta}^s_{20} = C_A\left(\frac{11}{3} - \frac{\epsilon}{3}\right) - \frac{2}{3}N_F, \qquad \bar{\beta}^e_{11} = 6C_F, \qquad \bar{\beta}^e_{02} = C_A(2-2\epsilon) - C_F(4-2\epsilon) - N_F,$$
$$\bar{\beta}^s_{30} = C_A^2\left(\frac{34}{3} - \frac{14}{3}\epsilon\right) - C_A N_F\left(\frac{10}{3}\right) - 2C_F N_F, \qquad \bar{\beta}^s_{21} = C_F N_F(2\epsilon). \tag{A7c}$$

In Eq. (A7b), the renormalized couplings are set equal, i.e. $\alpha_e = \alpha_s$. For the calculations in the off-shell case, also a UV renormalization of the external quark and gluons fields and the gauge parameter is needed. The corresponding renormalization constants can be found in Refs. [26,46].

According to operator renormalization in BM, the first perturbative coefficients of the UV-renormalized form factors in the FDH scheme are given by

$$\bar{\mathcal{F}}^{\mathrm{BM}}_{q,\mathrm{J}} = (1 + \delta\bar{Z}^{\mathrm{BM},(1)}_{\overline{\mathrm{MS}}} + \delta\bar{Z}^{\mathrm{BM},(2)}_{\overline{\mathrm{MS}}})(1 + \delta\bar{Z}^{\mathrm{BM},(1)}_5 + \delta\bar{Z}^{\mathrm{BM},(2)}_5) \times (1 + \bar{F}^{\mathrm{BM},(1)}_{q,\mathrm{J}} + \bar{F}^{\mathrm{BM},(2)}_{q,\mathrm{J}})_{\mathrm{ren}} + \mathcal{O}(\alpha_s^3), \tag{A8a}$$

$$\bar{\mathcal{F}}_{q,\mathrm{G}} = \frac{(1 + \delta\bar{Z}^{(1)}_{\mathrm{GG}})(R^{(1)}_{q,\mathrm{G}} + \bar{R}^{(2)}_{q,\mathrm{G}})_{\mathrm{ren}} + (\delta\bar{Z}^{(1)}_{\mathrm{GJ}} + \delta\bar{Z}^{(2)}_{\mathrm{GJ}})(\bar{R}^{(0)}_{q,\mathrm{J}} + \bar{R}^{(1)}_{q,\mathrm{J}})}{R^{(1)}_{q,\mathrm{G}} + \delta\bar{Z}^{(1)}_{\mathrm{GJ}} R^{(0)}_{q,\mathrm{J}}} + \mathcal{O}(\alpha_s^2). \tag{A8b}$$

$$\bar{\mathcal{F}}_{g,\mathrm{G}} = (1 + \delta\bar{Z}^{(1)}_{\mathrm{GG}} + \delta\bar{Z}^{(2)}_{\mathrm{GG}})(1 + \bar{F}^{(1)}_{g,\mathrm{G}} + \bar{F}^{(2)}_{g,\mathrm{G}})_{\mathrm{ren}} + \delta\bar{Z}^{(1)}_{\mathrm{GJ}}(R^{(1)}_{g,\mathrm{J}}/R^{(0)}_{g,\mathrm{G}}) + \mathcal{O}(\alpha_s^3), \tag{A8c}$$

$$\bar{\mathcal{F}}^{\mathrm{BM}}_{g,\mathrm{J}} = (1 + \delta\bar{Z}^{\mathrm{BM}(1)}_5)(1 + \bar{F}^{\mathrm{BM}(1)}_{g,\mathrm{J}})_{\mathrm{ren}} + \mathcal{O}(\alpha_s^2), \tag{A8d}$$

The subscript "ren" indicates that the coupling renormalization (4.1) is applied to the bare one-loop amplitudes. *After* UV renormalization, the evanescent coupling α_e is identified with the gauge coupling, i.e. $\alpha_e = \alpha_s$.

4. IR divergence structure

The IR divergence structure of one- and two-loop FDH amplitudes has been investigated in Ref. [24]. Specifying to the case of massless form factors with two external quarks and gluons, respectively, a $\mathbf{Z}$ factor subtracting all IR divergences is given by

$$\begin{aligned}\ln\mathbf{Z} =& \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\bar{\Gamma}'_{10}}{4\epsilon^2} + \frac{\bar{\Gamma}_{10}}{2\epsilon}\right) + \left(\frac{\alpha_e}{4\pi}\right)\left(\frac{\bar{\Gamma}'_{01}}{4\epsilon^2} + \frac{\bar{\Gamma}_{01}}{2\epsilon}\right)\\ &+ \left(\frac{\alpha_s}{4\pi}\right)^2\left(-\frac{3\bar{\beta}^s_{20}\bar{\Gamma}'_{10}}{16\epsilon^3} + \frac{\bar{\Gamma}'_{20} - 4\bar{\beta}^s_{20}\bar{\Gamma}_{10}}{16\epsilon^2} + \frac{\bar{\Gamma}_{20}}{4\epsilon}\right)\\ &+ \left(\frac{\alpha_s}{4\pi}\right)\left(\frac{\alpha_e}{4\pi}\right)\left(-\frac{3\bar{\beta}^e_{11}\bar{\Gamma}'_{01}}{16\epsilon^3} + \frac{\bar{\Gamma}'_{11} - 4\bar{\beta}^e_{11}\bar{\Gamma}_{01}}{16\epsilon^2} + \frac{\bar{\Gamma}_{11}}{4\epsilon}\right)\\ &+ \left(\frac{\alpha_e}{4\pi}\right)^2\left(-\frac{3\bar{\beta}^e_{02}\bar{\Gamma}'_{01}}{16\epsilon^3} + \frac{\bar{\Gamma}'_{02} - 4\bar{\beta}^e_{02}\bar{\Gamma}_{01}}{16\epsilon^2} + \frac{\bar{\Gamma}_{02}}{4\epsilon}\right).\end{aligned} \tag{A9a}$$

The relation between the perturbative coefficients of $\ln\mathbf{Z}$ and the UV-renormalized form factors is given by

$$\begin{aligned}(\ln\mathbf{Z})^{(1)} &= \bar{\mathcal{F}}^{\mathrm{BM},(1)}_{a,\mathrm{A}}\Big|_{\mathrm{poles}},\\ (\ln\mathbf{Z})^{(2)} &= \bar{\mathcal{F}}^{\mathrm{BM},(2)}_{a,\mathrm{A}}\Big|_{\mathrm{poles}} - \frac{1}{2}(\mathcal{F}^{\mathrm{BM},(1)}_{a,\mathrm{A}})^2\Big|_{\mathrm{poles}}.\end{aligned} \tag{A9b}$$

The $\mathbf{Z}$ factor is written in terms of the IR anomalous dimensions $\bar{\Gamma}'_{mn} = -2\bar{\gamma}^{\mathrm{cusp}}_{mn} C_{q/g}$ and $\bar{\Gamma}_{mn} = 2\bar{\gamma}^{q/g}_{mn}$ with $C_q = C_F$ for the quark form factor and $C_g = C_A$ for the gluon form factor. In FDH, the values of the partonic IR anomalous dimensions $\bar{\gamma}^{\mathrm{cusp}}_{mn}$, $\bar{\gamma}^q_{mn}$, and $\bar{\gamma}^g_{mn}$ are known up to the two-loop level [24]. Together with the known values of the one-loop β coefficients it is therefore possible to predict the entire IR divergence structure of the FDH form factors up to the two-loop level. Since Eq. (A9a) is written in terms of UV renormalized couplings, they can be set equal ($\alpha_e = \alpha_s$).

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